

MULTI-PERIOD PRODUCTION PLANNING IN FUZZY ENVIRONMENT

by

Yu-Long Liu

B.S., East-China Petroleum Institute, 1976

M.S., East-China Petroleum Institute, 1981

A MASTER'S THESIS
submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

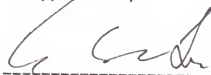
College of Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1989

Approved by:



Major Professor

LD
2668
174
IE
1989
L585
C.2



A11208 618010

ACKNOWLEDGEMENT

I would like to thank Dr. E. Stanley Lee for introducing me to the subject of "Multi-Stage Decision Making In Fuzzy Environment".

CONTENT

ACKNOWLEDGEMENT.....	i
TABLE OF CONTENT.....	ii
LIST OF FIGURES.....	iv
LIST OF TABLES.....	v
CHAPTER 1. INTRODUCTION.....	1
CHAPTER 2. A SUMMARY OF PREVIOUS RESEARCH.....	3
2-1. Multi-Period Production Planning Model.....	3
2-2. Multi-Stage Decision Making in Fuzzy Environment.....	6
2-2-1. Decision Making in Fuzzy Environment.....	7
2-2-2. Bellman and Zadeh's Fuzzy Multi-Stage Decision Processes.....	10
2-2-3. Some Extensions of Fuzzy Multi-Stage Decision Processes.....	14
2-3. The Application of Fuzzy Sets Theory in Production Planning.....	20
CHAPTER 3. THE GENERAL APPROACH BY THE USE OF FUZZY NUMBERS.....	28
3-1. A General Fuzzy Approach.....	28

3-2. Fuzzy Numbers.....	29
3-3. The Fuzzy Number Ranking Methods.....	30
CHAPTER 4. MULTI-PERIOD PRODUCTION PLANNING (MPPP)	
IN FUZZY ENVIRONMENT.....	36
4-1. The MPPP Problems.....	36
4-2. MPPP With Fuzzy Costs and Scalar Demand.....	39
4-3. MPPP With Fuzzy Demand and Scalar Costs.....	47
4-4. MPPP With Fuzzy Costs and Fuzzy Demand.....	50
4-5. Comparison of Three Fuzzy MPPP Methods.....	54
CHAPTER 5. OTHER APPLICATIONS OF THE GENERAL FUZZY	
APPROACH BY THE USE OF FUZZY NUMBERS.....	61
5-1. Introduction.....	61
5-2. The Network Problem.....	61
5-3. Fuzzy Resource Allocation Problem.....	65
5-4. The Taxi Driver's Best Strategy- An Example of Fuzzy Stochastic Processes.....	71
CHAPTER 6. CONCLUSIONS AND SUGGESTIONS.....	77
REFERENCES.....	79

LIST OF FIGURES

Figure 2-1. The Relation Between G and C and D.....	11
Figure 2-2. The Optimal Decision.....	11
Figure 3-1. The Membership Function of A Fuzzy Number...	31
Figure 3-2. A Triangular Fuzzy Number $M=(a,c,b)$	31
Figure 3-3. Lee-Li's Method.....	35
Figure 4-1. Cost Comparison of Two Different Schedules.....	45
Figure 5-1. The Network Problem.....	61
Figure 5-2. The Best Route.....	63
Figure 5-3. The Two Best Routes Obtained By Crisp Method.....	64
Figure 5-4. Comparison of Fuzzy Profits.....	70
Figure 5-5. The Taxi Driver's Best Strategy.....	76

LIST OF TABLES

Table 2-1. Lot-Sizing Procedures Used In The Experiments.....	5
Table 2-2. The Six Best Lot-Sizing Rules.....	6
Table 2-3. The State Transition Table.....	23
Table 4-1. The Original Data of Example 1.....	39
Table 4-2. The Translated Data of Example 1.....	40
Table 4-3. Schedule Obtained by The Crisp Model.....	41
Table 4-4. Schedule Obtained by The Fuzzy Model.....	44
Table 4-5. The Original Data of Example 2.....	47
Table 4-6. Schedule Obtained by The Fuzzy Model.....	49
Table 4-7. The Original Data of Example 3.....	51
Table 4-8. The Translated Data of Example 3.....	51
Table 4-9. Schedule Obtained by The Crisp Model.....	52
Table 4-10. Schedule Obtained by The Fuzzy Model.....	53
Table 4-11. Comparison of The Three Fuzzy MPPP Methods..	59
Table 5-1. The Original Data of Fuzzy Resource Allocation Problem.....	65
Table 5-2. The Optimal Proposals Obtained by The Fuzzy Method.....	68
Table 5-3. The Translated Data of Fuzzy Resource Allocation Problem.....	69
Table 5-4. The Optimal Proposals Obtained by The Crisp Method.....	69

MULTI-PERIOD PRODUCTION PLANNING IN FUZZY ENVIRONMENT

CHAPTER 1. INTRODUCTION

Production planning has a considerable impact on the economy of most firms, and many different production planning models have been developed [1]. Several multi-period production planning (MPPP) models have also been developed. One example is the well known Wagner-Whitin (W-W) model.

Because of its multi-period nature, production planning must be done before the beginning of production, yet the exact information needed is not available until the production events occur. So in MPPP problems, the future parameters must be estimated. A frequently used method is to give some interval estimates. For example, the management may estimate "the setup cost at period 3 will be approximately \$30,000, will be no more than \$40,000 and no less than \$10,000. " For almost all current models, these interval estimates must be translated into single numbers. This kind of translation may result not only in errors, but also in the loss of a considerable amount of information. Because fuzzy numbers can be used to overcome these difficulties, they offer an ideal approach to solving real-world MPPP problems.

The main purpose of this study is to find an effective method to solve the interval MPPP problems while retaining all the original information. Our approach uses a problem-oriented point of view. We let the problem that we are trying to solve determine the methods we apply to it, instead of letting our techniques determine what problems we must be willing and able to tackle.

In the following chapter, the previous work related to the subject is reviewed: (1) multiperiod production planning model, (2) fuzzy multistage decision making processes, and (3) application of fuzzy sets theory in production planning. Based on this survey, a more general fuzzy multi-stage decision approach by the use of fuzzy numbers is proposed. To introduce the nomenclature, a brief review of fuzzy number and fuzzy number ranking is presented in Chapter 3. In Chapter 4, three different fuzzy MPPP problems are solved. Chapter 5 considers other kinds of multi-stage decision making problems with fuzzy goals and fuzzy constraints defined in different spaces. Finally, in Chapter 6, we give brief concluding remarks and some suggestions for further studies.

CHAPTER 2. A SUMMARY OF PREVIOUS RESEARCH

In this chapter, a short summary of previous research will be given for the three subjects: multi-period production planning model, fuzzy multi-stage decision making processes, and applications of fuzzy sets theory in production planning.

2-1. Multi-Period Production Planning Model

Many production planning models, such as single-product network model [2], linear models [3-5] and quadratic cost model [6], have been developed. Dynamic programming [7], has been applied to solve MPPP problems under certain restricted assumptions [8-9]. W-W [10] and Wagner [11] have provided such a model for the case of a fixed plus variable production cost structure, where shortages are not allowed. Zangwill [12] has extended the W-W algorithm to the case where backorders are allowed. W-W's algorithm has also been extended to include capacity limitations [13-14] and dynamic demand [15].

William et al. [16] examined the effects of serially correlated demand on the performance of the following four well-known lot-sizing rules: economic order quantity [17-18], part period balancing [17,19], minimum cost per period/Silver-Meal [20], and W-W algorithm [10]. Their

simulation experiment results indicated the W-W algorithm was the best rule.

Bookbinder and H'ng [21] evaluated six separate lot-sizing methods for use in a rolling schedule. These methods include the W-W [10] algorithm, the Silver-Meal heuristic [20], Maximum Part-Period Gain of Karni [22], Heuristics 1 and 2 of Bookbinder and Tan [23], and Modification 1 to the Silver-Meal heuristic by Silver and Miltenburg [24]. They found that for larger forecast windows, with the horizon equal to or larger than 10 periods, "the most effective lot-sizing method was the Wagner-Whitin algorithm."

Wemmerlov [25] discussed lot-sizing in time-phased order point systems under three different conditions: no forecast errors present, forecast errors present but no safety stocks, and finally, with forecast errors present but with safety stocks introduced to counter the effects of the demand uncertainty. He observed fourteen different single stage lot-sizing procedures (see Table 2-1) during simulation experiments where each of these operating environments has been modeled. His results show that the W-W procedure is one of the six best rules (Table 2-2).

In addition, W-W's work is continually referred to in most recently published papers [29-31].

TABLE 2-1. [25]

Lot-Sizing Procedures Used In The Experiments

Procedure	Abbreviation	Reference
Wagner-Whitin	W-W	[10]
Economic Order Quantity	EOQ	[17,18]
Discrete Version of EOQ	EOQ-D	[26,18]
Part-Period Balancing	PPB	[17]
PPB with Look Ahead-Look Back	PPB w. LA-LB	[17]
Least Unit Cost	LUC	[17,18]
Order Moment	OM	[27,18]
Silver-Meal	SM	[20]
Groff's Rule	GMR	[28,18]
WMR1	WMR1	[18]
WMR2	WMR2	[18]
WMR3	WMR3	[18]
Period Order Quantity	POQ	[17,18]
Lot for Lot	LFL	[17]

From the above review, the W-W model appears to be a preferred model in solving the MPPP problem. However, all the methods require precise or stochastic data, while the data are typically given in imprecise terms. A better way to indicate these imprecise data is to use fuzzy

TABLE 2-2 [25]

The Six Best Lot-Sizing Rules, In Terms Of Setup And Holding Cost, In Environments With And Without Forecast Errors

Rank	Without Forecast Errors	With Forecast Errors
1	W-W	PPB
2	GMR	WMR3
3	SM	WMR2
4	WMR1	QM
5	PPB w. LA-LB	W-W
6	WMR3	EOQ

numbers. Thus, the production planning problem becomes a fuzzy problem. To solve the fuzzy MPPP problem, fuzzy multi-stage decision making processes will be used.

2-2. Multi-Stage Decision Making in Fuzzy Environment

Dynamic programming is an effective technique for solving decision making problems such as problems encountered in design, cluster analysis, pattern recognition and control theory for both the stochastic and adaptive types. Considerable advances have been made in its theoretical development and its computational aspects [32,33] as well as its application to real-life systems

[34,35].

The usefulness of fuzzy sets for decision making was recognized early. Many papers on fuzzy decision making were published [36-53] after Bellman and Zadeh (B-Z) in 1970 [54] introduced a general framework within which the analysis of a large spectrum of problems was made possible. In particular, it provided tools for studying multi-stage decision making processes. Other authors who have made noteworthy contributions in the area of fuzzy dynamic programming include Chang [55], Gluss [56], Kacprzyk [57,91], Nojiri [58], Stein [59], Esogbue & Ramesh [60], and Baldwin & Pilsworth [61]. Among these, B-Z's fuzzy multi-stage decision making process is the most basic and recognized one.

2-2-1. Decision Making in Fuzzy Environment [54]

There are many ways in which fuzziness appears in decision making, especially with regard to (a) the set of alternatives, (b) the set of constraints, (c) the choice of alternatives, and (d) the performance function which associates with each alternative return resulting from the choice of that alternative. We recapitulate them through the following definitions.

(1) Fuzzy Goal

Given $X = \{x\}$, a fuzzy goal, $G \subset X$ is a fuzzy subset of X , characterized by its membership function $\mu_G(x)$. For

instance, if $X: \mathbb{R} \rightarrow [0,1]$ (the real line) then the fuzzy goal which expects "x to be substantially larger than 10" may be subjectively represented by its membership function as:

$$\mu_G(x) = \begin{cases} 0, & x < 10 \\ (1 + (x-10)^{-2})^{-1}, & x \geq 10. \end{cases}$$

(2) Fuzzy Constraint

Given $X = \{x\}$, a fuzzy constraint, $C \subset X$ is a fuzzy subset of X which is characterized by the membership function $\mu_C(x)$. The fuzzy constraint requiring x "to be approximately between 2 and 10" could be represented by a fuzzy set whose membership function might be of the form:

$$\mu_C(x) = (1 + a(x-6)^m)^{-1},$$

where a is a positive number and m is a positive even integer chosen in such a way as to reflect the sense in which the approximation to the interval $[2, 10]$ is to be understood.

(3) Fuzzy Decision

The fuzzy decision making problem in a fuzzy environment can be stated as "Attain G and satisfy C ". That is, the fuzzy decision D is a fuzzy set in X resulting from some aggregation (or confluence) of G and C , i. e.

$$D = G * C \quad (2-1)$$

or, in terms of their constituent membership functions

$$\mu_D(x) = \mu_G(x) * \mu_C(x) \quad (2-2)$$

where the aggregation $*$ is some operation between two fuzzy sets.

Among the operations, the most important one is intersection, that is

$$D = G \wedge C \quad (2-3)$$

or, in terms of their constituent membership functions and the fuzzy set theoretical interpretation of the intersection operation,

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x) = \min \{ \mu_G(x), \mu_C(x) \} \quad (2-4)$$

This relation between G and C is depicted in Fig.2-1.

More generally, suppose that we have n goals, G_1, G_2, \dots, G_n and the given constraints C_1, C_2, \dots, C_m . That is,

$$D = G_1 \wedge G_2 \wedge \dots \wedge G_n \wedge C_1 \wedge C_2 \wedge \dots \wedge C_m, \quad (2-5)$$

and correspondingly

$$\mu_D(x) = \mu_{G_1}(x) \wedge \mu_{G_2}(x) \wedge \dots \wedge \mu_{G_n}(x) \wedge \mu_{C_1}(x) \wedge \mu_{C_2}(x)$$

$$\hat{\mu}_{G_1}(x) \wedge \dots \wedge \mu_{G_m}(x) \quad (2-6)$$

We note that since both G and C are convex fuzzy sets, then D is also convex in the fuzzy sense. This definition may be interpreted to mean that a decision is:

decision = confluence of goals and constraints.

(4) Optimal decision

The optimal decision is defined as such $x \in X$ that

$$\mu_D^*(x) = \max \mu_D(x). \quad (2-7)$$

For example, let the membership function of the fuzzy goal G "x should be much larger than 5" and fuzzy constraint "x should be more or less between 5 and 6" be as shown in Fig.2-2. The min-type optimal fuzzy decision is $x = 8$.

2-2-2. B-Z's Fuzzy Multi-stage Decision Processes [54]

All the classical forms of dynamic programming processes, such as discrete, continuous, deterministic, stochastic, and adaptive, can be fuzzified. For instance, the state, decision, transformation, and return function as well as the termination time can all be made fuzzy [54]. In other words, a completely fuzzy system operating in a fuzzy environment is described.

Consider the finite deterministic automation $A = (U, X, f)$, where $U = \{a_1, a_2, \dots, a_n\}$, $X = \{s_1, s_2, \dots, s_n\}$ are finite sets called the control and state spaces,

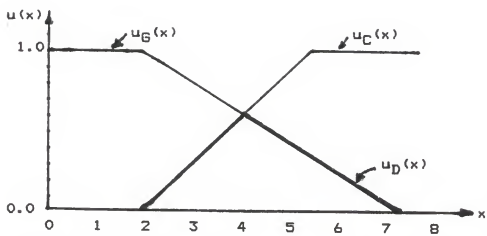


Figure 2-1. The relation between G and C and D.

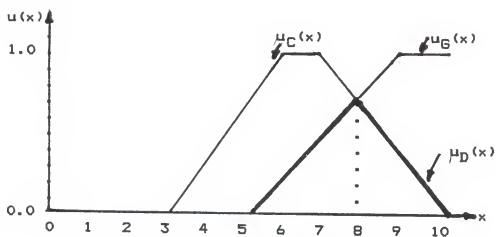


Figure 2-2. The optimal decision.

respectively, and $f: X \times U \rightarrow X$. The temporal evolution of A is described by the state equation

$$x_{t+1} = f(x_t, u_t) \quad t = 0, 1, \dots, N-1, \quad (2-8)$$

where $x_0 \in X$ is the initial state and N is the final time which we assume to be fixed. A fuzzy control constraint, C^t , is a fuzzy subset of U defined by the membership function $\mu_t(u_t)$ and a fuzzy goal, G^N , is a fuzzy subset of X defined by the membership function $\mu_{G^N}(x)$. Given an initial state x_0 , we are interested in finding a maximizing decision via dynamic programming.

We can at once express the decision, a decomposable fuzzy set in $U \times U \times \dots \times U$ as:

$$R = C^0 \wedge C^1 \wedge \dots \wedge C^{N-1} \wedge G^N, \quad (2-9)$$

where G^N is the fuzzy set in $UXUX \dots XU$ which induces G^N in X . In terms of membership functions, we have:

$$\mu_D(u_0, u_1, \dots, u_{N-1}) = \min (\mu_{C^0}(u_0), \mu_{C^1}(u_1), \dots,$$

$$\mu_{C^{N-1}}(u_{N-1}), \mu_{G^N}(x_N)), \quad (2-10)$$

where x_N is expressible as a function of x_0 and u_0, \dots, u_{N-1} .

We may rephrase the problem as: find the sequence of inputs u_0, \dots, u_{N-1} which maximizes μ_D of above equation.

The solution may be conveniently expressed in terms of π_t , the policy function with

$$u_t = \pi_t(x_t), \quad t = 0, 1, 2, \dots, N-1. \quad (2-11)$$

Dynamic programming may then be employed to obtain both the u_t , and the maximizing decisions u_0^M, \dots, u_{N-1}^M .

More specifically, this reduces to:

$$\begin{aligned} \mu_D(u_0^M, \dots, u_{N-1}^M) = \max_{u_0, \dots, u_{N-2}} \max_{u_{N-1}} (\mu_0(u_0) \wedge \dots \\ \wedge \mu_{N-2}(u_{N-2})) \wedge u_{N-1}(u_{N-1}) \wedge u_{GN}(f(x_{N-1}, u_{N-1})) \end{aligned} \quad (2-12)$$

Now, if r is a constant and g is any function of u_{N-1} , we have the identity:

$$\max_{u_{N-1}} (r \wedge g(u_{N-1})) = r \wedge \max_{u_{N-1}} g(u_{N-1}). \quad (2-13)$$

Consequently, (2-12) may be rewritten as:

$$\begin{aligned} u_D(u_0^M, \dots, u_{N-1}^M) = \max_{u_0, \dots, u_{N-1}} (u_0(u_0) \wedge \dots \wedge u_{N-2}(u_{N-2}) \\ \wedge u_{GN-1}(x_{N-1})), \end{aligned} \quad (2-14)$$

where

$$u_{GN-1}(x_{N-1}) = \max_{u_{N-1}} (u_{N-1}(u_{N-1}) \wedge u_{GN}(f(x_{N-1}, u_{N-1}))) \quad (2-15)$$

may be regarded as the membership function of a fuzzy goal at time $t = N - 1$ which is induced by the given goal G^N at time $t=N$.

On repeating this backward iteration, which is a simple instance of dynamic programming, we obtain the set of recurrence equations:

$$\mu_{GN-v}(x_{N-v}) = \max (\mu_{N-v}(u_{N-v}) \wedge \mu_{GN-v+1}(x_{N-v+1})),$$

$$x_{N-v+1} = f(x_{N-v}, u_{N-v}), \quad v=1, \dots, N. \quad (2-16)$$

which yield the solution of the problem. Thus, a maximizing decision u_0^M, \dots, u_{N-1}^M is given by the successive maximizing value of u_{N-v} in (2-16), with u_{N-v}^M defined as a function of x_{N-v} , $v=1, \dots, N$.

2-2-3. Some Extensions of Fuzzy Multi-Stage Decision Processes

Some extensions of B-Z's fuzzy multi-stage decision making processes have been developed and are briefly discussed as follows.

(1) Fuzzy stochastic dynamic programs [54]

The fuzzy dynamic program is assumed to be a Markov chain whose state transitions are governed by a conditional probability function $p(x_{t+1}; x_t, u_t)$, $t = 0, 1, \dots$, specifying the probability of attaining a state $x_{t+1} \in X = \{s_1, \dots, s_n\}$ from a state $x_t \in X$ and under a control $u_t \in U = \{c_1, \dots, c_m\}$. At each control stage t , the control u_t is subject to a fuzzy constraint $\mu_C(u_t)$, $t = 0, \dots, N-1$, and on the final state x_N a fuzzy goal

$\mu_{GN}(x_N)$ is imposed. Since the system under control is stochastic, the value of the membership function of the fuzzy decision is a random variable. We may evidently no longer state the problem as to find an optimal sequence of controls maximizing the membership function of the fuzzy decision. Rather, Some expectation should be involved as is usually done in handling problems with random elements.

There are two basic problem formulations:

(A) The formulation of B-Z [54]

Find an optimal sequence of controls u_0^*, \dots, u_{N-1}^* maximizing the probability of attainment of the fuzzy goal subject to the fuzzy constraints, i.e.

$$\mu_D(u_0^*, \dots, u_{N-1}^*; x_0) = \max_{u_0, \dots, u_{N-1}} (\mu_{C0}(u_0) \wedge \dots \wedge \mu_{CN-1}(u_{N-1})) \wedge \mu_{GN}(x_N) \quad (2-17)$$

where

$\mu_{GN}(x_N)$ = The probability of attainment of the fuzzy goal, a function of x_{N-1} and u_{N-1} .

(B) The formulation of Kacprzyk and Staniewski [62]

Find an optimal sequence of controls u_0^*, \dots, u_{N-1}^* maximizing the expected value of membership function of the fuzzy decision, i.e.

$$\begin{aligned}
u_D(u_0^*, \dots, u_{N-1}^*; x_0) &= \max_{u_0, \dots, u_{N-1}} E u_D(u_0, \dots, u_{N-1}; x_0) \\
&= \max_{u_0, \dots, u_{N-1}} E(u_{C0}(u_0) \wedge \dots \wedge u_{CN-1}(u_{N-1}) \wedge u_{GN}(x_N))
\end{aligned}
\tag{2-18}$$

Evidently in both cases we are in fact interested in finding an optimal strategy $A^* = (a_0^*, \dots, a_{N-1}^*)$.

(2) Fuzzy dynamic programming with implicitly specified termination time [54]

There are many problems in which the termination time is unknown in advance, or irrelevant, and we are mostly interested in attaining some final state; the process terminates when the state attains for the first time some specific value or enters some specific subset of the state space. The termination time is therefore implicitly defined by specifying that value or subset. That is, the termination time N of the fuzzy multi-stage decision making process is no longer fixed but is determined implicitly by the termination set. Note here T is non-fuzzy and that the fuzzy set G is defined in T , rather than in X . Then for $t=0$:

$$\begin{aligned}
u_D(u_0, \dots, u_{N-1}; x_0) &= \mu_C(u_0; x_0) \wedge \mu_D(u_1, \dots, \\
&\quad u_{N-1}; f(x_0, u_0))
\end{aligned}
\tag{2-19}$$

where the termination time N is also a function of x_0 and

u_0, u_1, \dots . Let the successive inputs u_0, u_1, \dots, u_{N-1} be determined by a stationary time invariant policy function $\pi, \bar{\pi}: T' \rightarrow U$, which associates with each state $x_t \in T'$ an input u_t which should be applied to A when it is in state x_t . Thus,

$$u_t = \pi(x_t), \quad t = 0, \dots, N-1; \quad x_t \in T'. \quad (2-20)$$

Also, given that we can write the membership function

$$\mu_D(x_0) = \mu_D(x_0; \pi),$$

$$\mu_C(u_0; x_0) = \mu_C(\pi(x_0); x_0),$$

$$\mu_D(u_1, \dots, u_{N-1}; f(x_0, u_0)) = (\mu_D(f(x_0, \pi(x_0); \pi))), \quad (2-21)$$

we then have:

$$\mu_D(x_0; \pi) = \mu_C(\pi(x_0); x_0) \wedge \mu_D(f(x_0, \pi(x_0); \pi)), \quad x_0 \in T'. \quad (2-22)$$

This is a system of L equations (one for each value of x_0) in the μ_D . This system determines μ_D as a function of x_0 for each . If $\mu_D=0$, there does not exist a finite N such that $x_N \in T$. Furthermore, $y = y$ for states in T .

(3) Fuzzy dynamic programs with fuzzy termination time [57,59]

If the termination time for attainment of our predetermined fuzzy goal G under fuzzy constraints C is fuzzy, the multi-stage decision process is called fuzzy

dynamic program with fuzzy termination time.

Let $u_T(t)$, $t \in T$, be the grade of membership indicating the degree of preference of t as the termination time. We assume that when $u_T=1$, we have the best preferred stopping time and that conversely $u_T=0$ indicates the impossibility or undesirability of stopping at this time. The goal to be attained, $G^t \in X$ may be modified as:

$$G^{t_k}_1 = u_T(t_k) G^{t_k} \quad (2-23)$$

where $t_k \in S(T)$, the support of T .

The problem then becomes: find an optimal termination time t_k^* and a maximizing decision $u_0^M, u_1^M, \dots, u_{k-1}^*$ such that:

$$\begin{aligned} u_D(u_0^M, u_1^M, \dots, u_{k-1}^M) &= \max_{t_{ku}^*} \{ \mu_{C_0}(u_0) \wedge \mu_{C_1}(u_1) \wedge \dots, \\ &\quad \wedge \mu_{C^{t_{k-1}}}(u_{t_{k-1}}) \wedge \mu_{G^{t_k}}(x_{t_k}) \} \end{aligned} \quad (2-24)$$

For the detailed fuzzy dynamic programming formulation for solving this problem, refer to Stein's work [59].

(4) Fuzzy dynamic programming under fuzzy mappings

Baldwin and Pilsworth [61] derived a dynamic programming functional equation for a multi-stage decision process in which the state mapping from one stage to the next is defined by a fuzzy automata acting in a fuzzy environment where both control constraints and stage goals are fuzzy. They defined the fuzzy mapping as following:

A fuzzy mapping $f: X \rightarrow Y$ is a fuzzy set on $X \times Y$ with membership function $\mu_f(x, y)$.

A fuzzy function $f(x)$ is a fuzzy set on Y with membership function $\mu_{f(x)}(y) = \mu_f(x, y)$.

Let A be a fuzzy subset on X defined by membership function $\mu_A(x)$.

The fuzzy set $f(A)$ on Y is a fuzzy mapping of a fuzzy set defined as

$$\mu_{f(A)}(y) = \bigvee (\mu_A(x) \wedge \mu_f(x, y)); \quad \text{all } y \in Y, x \in X \quad (2-25)$$

where \wedge stands for MIN and \bigvee for MAX.

In their extension, a modified objective criterion is used. That is, a "truth function" is defined which, broadly speaking, represents the truth that the goals and constraints are satisfied. Their final equation reduces to B-Z's treatment for the case of nonfuzzy state mappings if the transition function f is made nonfuzzy.

Obviously, the basic idea of these models is B-Z's fuzzy multi-stage decision process. We should note that "so far, we have restricted our attention to situations in which the goals and the constraints are fuzzy sets in X , the space of alternatives. A MORE GENERAL CASE WHICH IS OF PRACTICAL INTEREST IS ONE IN WHICH THE GOALS AND THE CONSTRAINTS ARE FUZZY SETS IN DIFFERENT SPACES [54]."

2-3. The Application of Fuzzy Sets Theory in Production Planning

Fuzzy dynamic programming, even though still very young, has already been applied to production planning and inventory control [63-69], integrated regional development and R & D management control systems[70-74]. Two other important papers on the application of fuzzy sets theory in production planning are [75] and [76]. In consideration of the purpose of our study, Sommer [63] and Kacprzyk & Stainewski's work [68] are worthy to be introduced here.

(1) Sommer's work

Sommer [63] used fuzzy dynamic programming in solving a production planning problem. His problem and model are briefly described as follows:

The management of a company wants to close a certain plant within a definite time interval. Therefore production levels should decrease to zero as steadily as possible. The demand is assumed to be deterministic.

In Sommer's application, the state variable, x_t , is the inventory level at the beginning of period t and the decision variable, u_t , is the production level in period t . The objective in this problem is to maximize the achievement of two goals: "keeping the production as level as possible" (for u_t) and "having an ending inventory level close to zero" (for x_t). This achievement

measurement will be made through the use of normalized membership function for both u_t and x_t .

The transition function linking the state variables is:

$$f(x_{T-v}, u_{T-v}) = x_{T-v+1} = x_{T-v} + u_{T-v} - a_{T-v} \quad (2-26)$$

where T is the time horizon in number of periods, v is the stage number and a_{T-v} is the given deterministic demand for period $T-v$.

The recursive return function, which evaluates the achievement of the goals in terms of the membership functions of the variables, is:

$$\mu_{GT-v}(x_{T-v}) = \sup \{ \mu_{T-v}(u_{T-v}) \wedge \mu_{GT-v+1}(x_{T-v+1}) \} \quad (2-27)$$

The problem is thus solved using the B-Z's fuzzy multi-stage decision process.

(2) Kacprzyk and Straszak's study

Kacprzyk and Straszak [68] have developed a procedure for determining the optimal time-invariant (long term) production planning model given fuzzy inputs, by also using the B-Z's fuzzy dynamic programming approach. This paper considers the determination of optimal inventory policy of firms from a global viewpoint of top management. The inventory is represented as a fuzzy system with the

fuzzy inventory level as the output, the fuzzy replenishment as the input and fuzzy demand. The control problem is formulated in terms of decision making in a fuzzy environment with fuzzy constraints imposed on replenishments, a fuzzy goal of preferable inventory levels to be attained and the fuzzy decision as the intersection of fuzzy constraints and the fuzzy goal at subsequent stages. The planning horizon is finite. The problem is to find an optimal time-invariant strategy relating the optimal replenishments to the current inventory levels, maximizing the membership function of fuzzy decision.

For the problem considered, the inventory is represented by the fuzzy system under control. Z_t is the fuzzy inventory level at time t , $Z_t \in F(U)$, $U = \{c_1, \dots, c_m\}$; and D is the fuzzy demand, $D \in F(Y)$, $Y = \{d_1, \dots, d_p\}$.

The temporal evolution is given by

$$Z_{t+1} = Z_t + R_t - D. \quad (2-28)$$

Since the dimensions of the $F(\cdot)$'s may be high, they use approximated reference fuzzy sets and specify $F(X) = \{S_1, \dots, S_r\}$ and $F(U) = \{C_1, \dots, C_s\}$, where r and s are sufficiently small numbers.

The dynamics of the inventory is now given by

$$Z_{t+1} = A(Z_t + R_t - D) \quad (2-29)$$

which may conveniently be represented by the following table:

Table 2-3.

		Z_t	
		$S_1 \dots \dots \dots S_r$	
R_t	C_1	Z_{t+1}	
	.		
	.		
	C_s		

That is, for each reference fuzzy replenishment at time t and reference fuzzy inventory at time t the resulting reference fuzzy inventory at time $t+1$ is given.

To determine the optimal time-invariant strategy, they provide the following policy-iteration-type algorithm.

Step 1. Choose an arbitrary time-invariant strategy $a_{\#}$.

Step 2. Solve the following vector equation:

$$\begin{aligned} \mu_D(a_{\#}; Z) &= \mu_C(a(Z); Z) \wedge \mu_G(A(Z+a(Z)-D)) \\ &\quad \wedge b\mu_D(a_{\#}; A(Z+a(Z)-D)) \end{aligned} \quad (2-30)$$

The unknowns are the u_D 's. If their values are not unique, as often occurs, then one has to take the greatest values possible, which results from the nature of ' \wedge ' operation.

Step 3. Improve the strategy. Using the u_D 's determined, seek for each $i \in \{1, \dots, r\}$ a $z' \in V$, such that

$$z' = \arg \min_V (\mu_C(z(S_i); i) \wedge \mu_B(A(S_i + z(S_i) - D)) \wedge \mu_D(a_{\#}(S_i + a(S_i) - D))) \quad (2-31)$$

Step 4. If the strategy determined is the same as the previous one, then it is optimal. Otherwise, return to Step 2.

Finally, the optimal strategy will be indicated by the fuzzy conditional statement. For example, suppose the particular reference fuzzy inventory levels S_1, \dots, S_t are related through the policy function to the reference fuzzy replenishments C_1, \dots, C_s , e.g. as follows:

$$C_U = a(S_1), \quad C_W = a(S_2), \dots, \quad C_V = a(S_t),$$

then these replenishment rules are arranged as follows:

```
IF (Z=S1) THEN (R=CU) ELSE
IF (Z=S2) THEN (R=CW) ELSE...
.
.
.
IF (Z=Sr) THEN (R=CV).
```

A very simple numerical example was also given in their paper with $X = \{1, \dots, 10\}$, $Y = \{1, \dots, 5\}$, $r=5$, $s=3$. The demand is treated as constant. Even for this simple problem, the operations involved in approximations and calculations are very tedious work.

Esogbue and Bellman [69] review some applications of

fuzzy dynamic programming and discuss fuzzy dynamic programming applications to multicriteria optimization, terminating and nonterminating control processes, cluster analysis, energy systems, water resources, integrated regional development, cancer research and resource allocation processes in general. They point out:

"Dynamic programming and its application to multi-stage decision processes of stochastic and adaptive type is well known. HOWEVER, ONLY A BEGINNING HAS BEEN MADE TO APPLY DYNAMIC PROGRAMMING TO FUZZY SYSTEMS." In our view, the main reason that fuzzy dynamic programming not been widely used is its tight restrictions.

One of its restrictions is same-space. B-Z's fuzzy multi-stage decision process requires the problem's fuzzy goal and fuzzy constraints to be fuzzy sets defined in the same space. In the fuzzy MPPP problem, we try to obtain a schedule which minimizes the total cost subject to the condition of satisfying the demand within the planning horizon. The alternatives and constraints are the units of production at each period, and the goal is to minimize the total cost. Obviously, the production planning problem's goal and constraints are fuzzy sets defined in different spaces. Although theoretically these different spaces can be reduced to the case where they are defined in the same space [54], this reduction cannot be easily

made.

The second major restriction is the computational requirement for large dimensional problems. In the real-world MPPP problem, the dimensions of F may be very high and so make the practical computation impossible. Kacprzyk and Straszak [68] have proposed using approximated reference fuzzy sets to overcome this dimensionality difficulty. This reference fuzzy sets approach is still limited and also may induce erroneous information or loss of original information.

As early as 1970 when Bellman and Zadeh developed the fuzzy multi-stage decision making processes, they stated that:

"As an application of the concepts introduced in the preceding sections, we shall consider a few basic types of problems involving multistage decision-making in a fuzzy environment. IT SHOULD BE STRESSED THAT, IN WHAT FOLLOWS, OUR MAIN PURPOSE IS TO ILLUSTRATE THE USE OF THE CONCEPTS OF FUZZY GOAL, FUZZY CONSTRAINT AND FUZZY DECISION, RATHER THAN TO DEVELOP A GENERAL THEORY OF MULTI-STAGE DECISION PROCESSES IN WHICH FUZZINESS ENTERS IN ONE WAY OR ANOTHER."

Thus, there exists no effective technique to solve our fuzzy MPPP problem. A new approach is needed to attack

this difficult problem.

CHAPTER 3. A GENERAL APPROACH

BY THE USE OF FUZZY NUMBERS

3-1. A General Fuzzy Approach

As discussed above, the crisp dynamic programming production planning model cannot guarantee an optimal solution due to the requirement of single-number representation. Further, fuzzy dynamic programming is not suited to solve practical MPPP problems. To avoid these difficulties, we propose a more general fuzzy approach by the use of fuzzy numbers, as follows:

- (1) employing appropriate fuzzy numbers to represent the estimated interval in the multi-stage decision problems;
- (2) using the operations of fuzzy numbers combined with dynamic programming to solve the problem;
- (3) determining the required minimum or required maximum fuzzy number by the use of fuzzy number ranking techniques.

The only difficulty is that at each stage the resulting values are fuzzy numbers. Since fuzzy numbers are frequently partial order, a meaningful fuzzy ranking technique must be used.

In the remaining sections of this chapter, we will introduce the basic knowledge on fuzzy numbers related to the above decision processes.

3-2. Fuzzy Numbers [87]

Generally speaking, a fuzzy number is a convex and normal fuzzy subset of the real line R , and its membership function must be piecewise continuous.

Mathematically, a fuzzy number A 's membership function can be indicated by

$$\mu_A(x) = \begin{cases} f_1(x), & \text{if } x_1 \leq x \leq x_2 \\ 1, & \text{if } x_2 \leq x \leq x_3 \\ f_2(x), & \text{if } x_3 \leq x \leq x_4 \\ 0, & \text{elsewhere} \end{cases} \quad (3-1)$$

and shown as in Fig.3-1.

Triangular fuzzy numbers [77] are especially suited to represent the interval estimates generally used in engineering practice. It not only represents the interval, but also give a clear representation of the possibility. In the example given in Chapter 1, the management really means that the possibility of \$30,000 is the highest and this possibility decreases as the estimated cost moves away from this highest possible point. Notice that the triangular fuzzy number exactly gives this representation. Thus, it can be represented as:

$$\mu_{A3}(x) = \begin{cases} x/20000 - 10000/20000 & 10,000 \leq x \leq 30,000 \\ 1 & x = 30,000 \\ (40000-x)/10000 & 30,000 \leq x \leq 40,000 \end{cases}$$

or simply

$$A_3 = (10,000, 30,000, 40,000).$$

In general, a triangular fuzzy number can be represented as:

$$\mu_A(x) = \begin{cases} (x-x_1)/(x_2-x_1) & x_1 \leq x \leq x_2 \\ 1 & x = x_2 \\ (x_3-x)/(x_3-x_2) & x_2 \leq x \leq x_3 \\ 0 & \text{elsewhere} \end{cases} \quad (3-2)$$

A simple notation $A = (a, c, b)$ with $a=x_1$, $c=x_2$ and $b=x_3$ is used to indicate a triangular fuzzy number (Figure 3-2).

The operations on fuzzy numbers are based on the conception of the α -cut fuzzy set. They are introduced in detail in [87].

3-3. The Fuzzy Number Ranking Methods

Because fuzzy number is frequently partial order, it cannot be easily compared. This is one reason that the approach based on the use of fuzzy numbers is not widely applied. Many researchers have developed different methods [78-86], but almost every method has its drawback or limitation [88]. Most recently, Lee-Li [89-90] proposed using generalized mean value and standard deviation based on the probability measure of fuzzy events to rank fuzzy numbers. Lee-Li's method, although it has its own

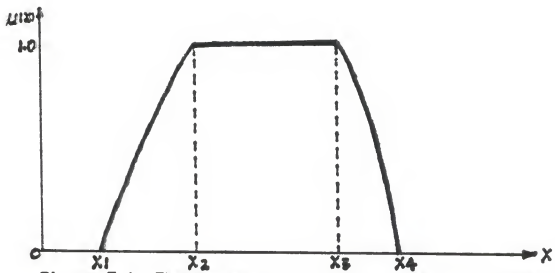


Figure 3-1. The Membership Function of a Fuzzy Number

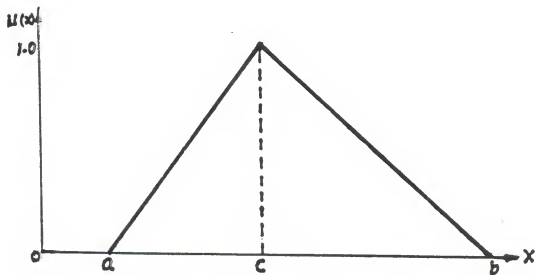


Figure 3-2. A Triangular Fuzzy Number $M=(a, c, b)$

in dealing with the nonnormal fuzzy sets, is probably the best one for ranking fuzzy numbers.

Lee-Li's method assumes two kinds of probability distributions for fuzzy events and derives the corresponding indices as follows:

(1) Uniform Distribution:

$$f(A_i) = 1/|I_i|, \quad A_i \subset U$$

$$M_U(A_i) = \frac{\int_{\mathbb{R}_{A_i}} x \cdot \mu_{A_i}(x) dx}{\int_{\mathbb{R}_{A_i}} \mu_{A_i}(x) dx} \quad (3-3)$$

$$G_U(A_i) = \left[\frac{\int_{\mathbb{R}_{A_i}} x^2 \cdot \mu_{A_i}(x) dx}{\int_{\mathbb{R}_{A_i}} \mu_{A_i}(x) dx} - M_U^2(A_i) \right]^{1/2} \quad (3-4)$$

(2) Proportional Distribution:

$$f(A_i) = C \cdot \mu_{A_i}(x)$$

$$M_P(A_i) = \frac{\int_{\mathbb{R}_{A_i}} x^2 \cdot \mu_{A_i}(x) dx}{\int_{\mathbb{R}_{A_i}} \mu_{A_i}^2(x) dx} \quad (3-5)$$

$$G_p(A_i) = \left[\frac{\int_{S_{A_i}} x^2 \cdot \mu_{A_i}^2(x) dx}{\int_{S_{A_i}} \mu_{A_i}(x) dx} M_p^2(A_i) \right]^{1/2} \quad (3-6)$$

where $f(A_i)$ = the probability density function;

C = a proportionality constant;

$M(A_i)$ = the generalized mean value of fuzzy sets;

$G(A_i)$ = the standard deviation of fuzzy sets;

u, p = subscripts denoting the uniform and proportional distributions respectively.

In the case of triangular fuzzy numbers, the indices reduce to

$$M_u(A_i) = 1/3 (a+b+c)$$

$$G_u(A_i) = 1/18 (a^2 + b^2 + c^2 - ab - ac - bc)$$

$$M_p(A_i) = 1/4 (a + b + c)$$

$$G_p(A_i) = 1/80 [(a-b)^2 + 2(a-c)^2 + 2(b-c)^2]$$

where $a = \inf S_i$.

$$b = \sup S_i.$$

$$\mu_A(c) = 1.$$

The two groups of indices are shown in Fig.3-3. Note that the mean value based on the proportional probability distribution is closer to the mode of the fuzzy number and the corresponding variance is smaller, which means that a stronger central tendency exists in the proportional case because of emphasizing the correlativity between possible

and probable.

In accordance with Lee-Li's rule, the ordering of two fuzzy sets is defined as that if

$$M(A_i) > M(A_j)$$

or

$$M(A_i) = M(A_j) \quad \text{and} \quad G(A_i) < G(A_j)$$

then

$$A_i > A_j$$

An example of comparing two triangular fuzzy numbers by Lee-Li's method is shown as following:

$$A = (3, 5, 8) \quad B = (3, 6, 7)$$

In uniform case

$$M_U(A) = 1/3 (3+5+8) = 5.33$$

$$G_U(A) = 1/18(3^2+5^2+8^2 - 3(5) - 3(8) - 5(8)) = 19/18$$

$$M_U(B) = 1/3 (3 + 6 + 7) = 5.33$$

$$G_U(B) = 1/18(3^2 + 6^2 + 7^2 - 3(6) - 3(7) - 6(7)) = 13/18$$

$$\text{Because } M_U(A) = M_U(B) \quad \text{and } G_U(A) > G_U(B)$$

then $A < B$.

In the proportional case

$$M_P(A) = 1/4(3+5+8)=4$$

$$M_P(B) = 1/4(3+6+7)=4$$

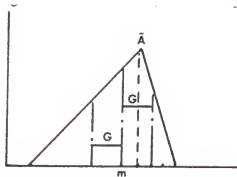
$$G_P(A) = 1/80 [(3-8)^2 + 2(3-5)^2 + 2(8-5)^2] = 51/80$$

$$G_P(B) = 1/80 [(3-7)^2 + 2(3-6)^2 + 2(7-6)^2] = 36/80$$

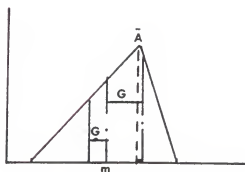
$$M_P(A) = M_P(B) \quad G_P(A) > G_P(B)$$

so that

$$A < B.$$



(a) "uniform"



(b) "proportional"

Figure 3-3. Lee-Li's method

CHAPTER 4.

MULTI-PERIOD PRODUCTION PLANNING IN A FUZZY ENVIRONMENT

4-1. The MPPP Problems

Let us assume that the production cost of period t is

$$K(P_t) = \begin{cases} 0 & \text{if } P_t = 0 \\ A_t + c_t P_t & \text{if } P_t > 0 \end{cases} \quad (4-1)$$

and the inventory holding cost of period t is

$$K(I_t) = h_t I_t \quad (4-2)$$

Then the general MPPP problem is:

$$\text{Minimize} \quad C_T = \sum_{t=1}^T (A_t + c_t P_t + h_t I_t)$$

Subject to

$$\begin{aligned} I_t &= I_{t-1} + P_t - D_t \\ 0 &\leq P_t \leq P_{\text{Max}} \\ 0 &\leq I_t \leq I_{\text{Max}} \\ t &= 1, 2, \dots, T-1, T. \end{aligned} \quad (4-3)$$

where

C_T = Total cost of production and inventory with the planning horizon $t = 1$ to T .

A_t = Fixed production cost in period t ;

c_t = Variable unit production cost at period t ;

h_t = the unit inventory holding cost for period t ;

P_t = Production in period t ;

P_{Max} = the maximum production capacity;

D_t = forecasted demand in period t ;

I_t = inventory at the end of period t ;

I_{Max} = the maximum inventory capacity.

This problem is usually solved by dynamic programming.

Due to the properties of optimal solution demonstrated by Wagner and Whitin [2]:

$$I_{t-1}P_t=0$$

$$P_t=0, D_t, D_t+D_{t+1}, D_t+D_{t+1}+D_{t+2}, \dots, D_t.$$

a more effective MPPP model by the use of dynamic programming was developed [2]. In this model, the total cost of production and inventory over the period j to k is:

$$\begin{aligned} C_{jk} &= k(P_{jk}) + k(I_{jk}) \\ &= A_{j+1} + c_{j+1}P_{j+1} + \sum_{t=j+1}^{k-1} h_t I_t \end{aligned}$$

where:

C_{jk} = cost of producing and inventory in period $j+1$ to satisfy demand in $j+1, j+2, \dots, k$.

The global optima can be obtained by the following dynamic programming recursion:

$$Z_k = \min_{0 \leq j \leq k-1} [Z_j + C_{jk}] \quad (k = 1, 2, \dots, T) \quad (4-5)$$

At each stage of the recursion, we seek to minimize the combination of the cost of production between two regeneration points (j and k) plus the optimal program up to j. The recursion is computed for k = 0 to T.

In the real-world MPPP problem, the amounts of A_t , C_t , h_t and D_t are estimated based on experience, expected value or other statistical techniques. These amounts must be approximated by single numbers when we solve the problem by crisp dynamic programming. This kind of approximation always loses some information. A better choice is to indicate the estimated amounts by the use of fuzzy numbers. In the following sections of this chapter, we will solve three MPPP problems with estimated costs and demands by applying the general fuzzy approach based on the use of fuzzy numbers to W-W model [2]. Specifically, we will develop a fuzzy MPPP model by: (1) using appropriate triangular fuzzy numbers to represent the interval estimates A_t , C_t , h_t and D_t in equation (4-4), (2) employing the fuzzy number operations '(+)' and '(.)' to carry out the corresponding operation '+' and '.' in equation (4-4) and (4-5) and, (3) applying Lee-Li's fuzzy number ranking method to complete the 'min' operation in equation (4-5). First, we will solve the

problems by both crisp model and fuzzy model, and then compare the results to see the advantage of our fuzzy MPFP approach.

3-2. MPPP with Fuzzy Costs and Scalar Demand

A firm is making a production plan for one kind of product over a planning horizon of three periods with no initial inventory. The information available is given in the following table.

TABLE 4-1.

period	demand	setup cost	unit cost	holding cost
t	D_t	A_t	c_t	h^t
	(unit)	(\$1,000)	(\$1000/unit)	(\$1000/unit.period)
2	30	(20,40,50)	(1, 3, 4)	1
3	30	(20,30,60)	(2, 3, 5)	2

In this table, the setup costs and variable unit costs are given by triangular fuzzy numbers which represent the interval estimates discussed in Chapter 3.

(1) Solution of dynamic programming production planning model

To obtain the optimal production schedule by crisp

dynamic programming production planning model, we have to translate the estimated numbers into single numbers and may obtain the following data.

TABLE 4-2.

t	D_t (unit)	A_t (\$1,000)	C_t (\$1,000)	h_t (\$1,000)
1	10	\$20	\$3	\$1
2	30	40	3	1
3	30	30	3	2

Notice that in the above table, we choose the most possible numbers as the approximated single numbers of the estimated intervals. Obviously, we lost all the other information.

In the process of solution, we follow the W-W approach [2].

$$Z_1 = C_{01} = A_1 + C_1(D_1) = 20 + 3(10) = 50$$

$$Z_1^* = 50$$

$$Z_2 = \begin{cases} Z_0 + C_{02} = A_1 + C_1(D_1 + D_2) + h_1(D_2) \\ \quad = 20 + 3(10 + 30) + 1(30) = 170 \\ Z_1^* + C_{12} = Z_1^* + A_2 + C_2(D_2) \\ \quad = 50 + 40 + 3(30) = 180 \end{cases}$$

$$Z_2^* = 170$$

$$\begin{aligned}
 & \left\{ \begin{aligned}
 Z_0 + C_{03} &= A_1 + C_1(D_1 + D_2 + D_3) + h_1(D_2 + D_3) + h_2(D_3) \\
 &= 20 + 3(10 + 30 + 30) + 1(30 + 30) + 1(30) \\
 &= 320 \\
 Z_3 &= \begin{cases}
 Z_1^* + C_{13} = Z_1^* + A_2 + C_2(D_2 + D_3) + h_2(D_3) \\
 \quad = 50 + 40 + 3(30 + 30) + 1(30) \\
 \quad = 300 \\
 Z_2^* + C_{23} = Z_2^* + A_3 + C_3(D_3) \\
 \quad = 170 + 30 + 3(30) \\
 \quad = 290
 \end{cases}
 \end{aligned} \right.
 \end{aligned}$$

$$Z_3^* = 290$$

The minimum total cost is \$290,000. The production schedule (Table 4-3) is found by tracing the solution backward.

$Z_3^* = 290$ ----Produce 30 units in period 3 for period 3.

$Z_2^* = 170$ ----Produce 40 units in period 1 for both period 1 and 2.

TABLE 4-3.

Period	1	2	3
Production	40	0	30

(2) Solution of fuzzy dynamic programming production planning model

Now let us employ fuzzy numbers to replace the

translated single numbers in our example and use the fuzzy general approach and W-W model to obtain the solution. We will use all of the information given in Table 4-1.

$$\begin{aligned} Z_1 &= C_{01} = A_1 (+) C_1 (.) D_1 \\ &= (15, 20, 50) (+) (2, 3, 6) (.) 10 \\ &= (35, 50, 110) \end{aligned}$$

$$Z_1^* = (35, 50, 110)$$

$$Z_2 = \begin{cases} Z_0 (+) C_{02} = A_1 (+) C_1 (.) (D_1 + D_2) + h_1(D_2) \\ \quad = (15, 20, 50) (+) (2, 3, 6) (.) 40 + 1(30) \\ \quad = (125, 170, 320) \\ Z_1 (+) C_{13} = Z_1 (+) A_2 (+) C_2 (.) D_2 \\ \quad = (35, 50, 110) (+) (20, 40, 45) (+) \\ \quad (+) (1, 3, 4) (.) 30 \\ \quad = (85, 180, 275) \end{cases}$$

Using Lee-Li's method,

$$\text{Mu}(125, 170, 320) = 205$$

$$\text{Mu}(85, 180, 275) = 180$$

$$180 < 205 \text{ ----- } (85, 180, 275) < (125, 170, 320)$$

$$Z_2^{\#} = Z_1^* (+) C_{13} = (85, 180, 275).$$

$$\begin{aligned}
 Z_0 (+) C_{03} &= A_1 (+) C_1 (.) (D_1 + D_2 + D_3) + h_1 (D_2 + D_3) + h_2 (D_3) \\
 &= (15, 20, 50) (+) (2, 3, 6) (.) 70 + 60 + 30 \\
 &= (245, 320, 560)
 \end{aligned}$$

$$\begin{aligned}
 Z_3 = Z_1^* (+) C_{13} &= Z_1^* (+) C_2 (.) (D_2 + D_3) + h_2 (D_3) \\
 &= (35, 50, 110) (+) (20, 40, 45) (+) \\
 &\quad (+) (1, 3, 4) (.) (30 + 30) (+) 1(30) \\
 &= (145, 300, 425)
 \end{aligned}$$

$$\begin{aligned}
 Z_2^* (+) C_{23} &= Z_2^* (+) A_3 (+) C_3 (.) D_3 \\
 &= (85, 180, 275) (+) (20, 30, 60) (+) \\
 &\quad (+) (2, 3, 5) (.) 30 \\
 &= (165, 300, 485)
 \end{aligned}$$

$$M_U(Z_0 (+) C_{03}) = M_U(245, 320, 560) = 375$$

$$M_U(Z_1 (+) C_{13}) = M_U(145, 300, 425) = 290$$

$$M_U(Z_2 (+) C_{23}) = M_U(165, 300, 485) = 316$$

$$Z_3^* = (145, 300, 425)$$

The production schedule (Table 4-4) is found by tracing the optimal fuzzy solution backward:

$Z_3^* = (145, 300, 425)$ ---- produce 60 units in period 2
for period 2 and 3;

$Z_1^* = (35, 50, 110)$ ----- produce 10 units in period 1
for period 1.

(3) Comparison of the solutions

The production schedules obtained by the crisp dynamic programming production planning model are different from

TABLE 4-4.

Period	1	2	3
Production (units)	10	60	0

the schedules obtained by the fuzzy production planning model. If the firm employs the production schedule obtained by the crisp model, the corresponding total cost should be (205, 290, 530), which is more than the total cost, (145, 300, 425) corresponding to the production schedule obtained by the fuzzy model (see Figure 4-1).

This difference is due to the different methods by which we deal with the estimated intervals. In the above example, it is more reasonable to indicate the estimated amount " A_1 is about \$20,000, no more than \$50,000, no less than \$15,000" by a fuzzy number $A_1 = (15000, 20000, 50000)$ than by a single number $A_1 = 20,000$. So there is some reason for us to believe that sometimes the production schedule obtained by fuzzy methods is better than the schedule obtained by the crisp method.

Further, according to the schedule obtained by the crisp model, the production manager will need exactly \$290,000. This is not true. To satisfy the forecasted demand of the planning horizon, the minimum total cost should be (205, 290, 530) thousand dollars. This fuzzy

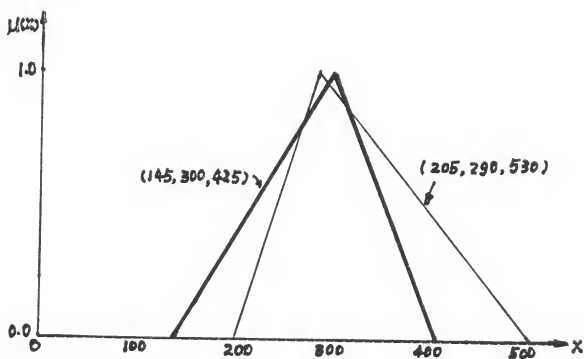


Figure 4-1. Cost Comparison of Two Different Schedules

cost means that the total production cost is about \$290,000, it might be as low as \$206,000 or might be as high as \$529,000. Having this idea in mind, the production manager will have a total structure of the cost when he makes his budget. He will not be confused when the real cost is much less or more than \$290,000.

Finally, we should realized that the schedule obtained by the crisp model and by the fuzzy model are not necessarily different. In this example, the fuzzy number operations we used are (a) addition of a scalar and a triangular fuzzy number; (b) addition of two triangular fuzzy numbers; and (c) multiplication of a triangular fuzzy number and a scalar. From [87], we know that if all the fuzzy costs in the problem are symmetrical triangular fuzzy numbers and the demands are scalars, then the results after they go through the above three kinds of operations are still symmetrical triangular fuzzy numbers. Recall that the main part of Lee-Li's fuzzy number ranking method is the generalized mean, and the generalized mean of any symmetrical triangular fuzzy number, $M=(a,c,b)$, is c . We may conclude that if all costs in the production planning problem are indicated by symmetrical triangular fuzzy numbers or scalars, then the schedule obtained by the fuzzy model should be the same as that obtained by the crisp model. So we can say that the

fuzzy model is an alternative to having to use single numbers in multi-period production scheduling. It can provide a comparable or better schedule than the crisp model.

4-3. MPPP With Fuzzy Demand and Scalar Costs

The problem is given by the data of Table 4-5. In this example, the forecasted demands are fuzzy numbers and the costs c_t , A_t and h_t are scalars.

TABLE 4-5.

t	D_t	A_t (\$1,000)	c_t (\$1,000)	h_t (\$1,000)
1	(5, 10, 20)	20	3	1
2	(25, 30, 40)	40	3	1
3	(20, 30, 40)	30	3	2

(1) Solution of Crisp Dynamic Programming

If we translate the estimated demands into single numbers and solve the problem by W-W model, the data will be the same as shown in Table 4-2, and the optimal schedule will be the same as showed in Table 4-3.

(2) Solution of Fuzzy Dynamic Programming

Here again, we combine the general fuzzy approach with

W-W model and use all of the information given in Table 4-5 to solve this MPPP problem.

$$\begin{aligned} Z_1 = C_{01} &= A_1 (+) c_1 (.) D_1 \\ &= 20 (+) 3 (.) (5, 10, 20) \\ &= (35, 50, 80) \end{aligned}$$

$$Z_1^* = (35, 50, 80)$$

$$\begin{aligned} Z_0 (+) C_{02} &= A_1 (+) c_1 (.) [D_1 (+) D_2] (+) h_1 (.) [D_1 (+) D_2 (-) D_1] \\ &= 20 (+) 3 (.) [(5, 10, 20) (+) (25, 30, 40)] (+) [(5, 10, 20) \\ &\quad (+) (25, 30, 40) (-) (5, 10, 20)] \\ &= (120, 170, 255) \end{aligned}$$

$$\begin{aligned} Z_2 = Z_1 (+) C_{12} &= Z_1 (+) A_2 (+) c_2 (.) D_2 \\ &= (35, 50, 80) (+) 40 (+) 3 (.) (25, 30, 40) \\ &= (150, 180, 240) \end{aligned}$$

$$M_u(Z_0 (+) C_{02}) = 545/3$$

$$M_u(Z_1 (+) C_{12}) = 570/3$$

$$Z_2^* = (Z_0 (+) C_{02}) = (120, 170, 255)$$

$$\begin{aligned} Z_0^* (+) C_{03} &= A_1 (+) c_1 (.) [D_1 (+) D_2 (+) D_3] (+) h_1 (.) [D_1 (+) D_2 (+) D_3 - D_1] \\ &\quad (+) h_2 (.) [D_1 (+) D_2 (+) D_3 (-) D_1 (-) D_2] \\ &= 20 (+) 3 (.) [(5, 10, 20) (+) (25, 30, 40) (+) (20, 30, 40)] \\ &\quad (+) [(5, 10, 20) (+) (25, 30, 40) (+) (20, 30, 40) (-) \\ &\quad (5, 10, 20)] (+) [(5, 10, 20) (+) (25, 30, 40) (+) (20, 30, 40)] \end{aligned}$$

$$\begin{aligned}
& (-) (5, 10, 20) (-) (25, 30, 40)] \\
& = (190, 330, 485) \\
Z_3^* = & Z_1^* (+) C_{13} \\
& = Z_1^* (+) A_2 (+) C_2 (.) [D_2 (+) D_3] (+) h_2 (.) [D_2 (+) D_3 (-) D_2] \\
& = (35, 50, 80) (+) 40 (+) 3 (.) [(25, 30, 40) (+) (20, 30, 40)] \\
& \quad (+) [(25, 30, 40) (+) (20, 30, 40) (-) (25, 30, 40)] \\
& = (215, 300, 415) \\
& Z_2^* (+) C_{23} \\
& = Z_2^* (+) A_3 (+) C_3 (.) D_3 \\
& = (120, 170, 255) (+) 30 (+) 3 (.) (20, 30, 40) \\
& = (210, 290, 405)
\end{aligned}$$

$$M_u(Z_0^* (+) C_{03}) = 1025/3$$

$$M_u(Z_1^* (+) C_{13}) = 930/3$$

$$M_u(Z_2^* (+) C_{23}) = 905/3$$

$$Z_3^* = (Z_2^* (+) C_{23}) = (210, 290, 405)$$

The production schedule (Table 4-6) is found by tracing the optimal fuzzy solution backward:

TABLE 4-6.

Period	1	2	3
Production (units)	(30, 40, 60)	0	(20, 30, 40)

$$Z_3^* = (Z_2^* (+) C_{23}) = (210, 290, 405) \text{ --- produce } D_3 = (20, 30, 40) \text{ units in period 3 for period 3;}$$

$$Z_2^* = (Z_0^* (+) C_{02}) = (120, 170, 255) \text{ --- produce } D_1 (+) D_2$$

units in period 1 for both period 1 and 2.

(3) Comparison of two solutions

In this example, the results obtained by the crisp model and by the fuzzy model are quite different. Firstly, the schedule obtained by the fuzzy model is represented by fuzzy numbers, which retain all the original information. But the schedule obtained by the crisp model is represented by single number, which lost most of the original information. The minimum total costs obtained by the crisp model and by the fuzzy model are similarly different.

These two differences are important and useful in practice. Suppose this example is a real problem. According to the schedule obtained by the crisp model, the firm must produce exactly 40 units in period 1 and 30 units in period 3. In practice, this is not necessary and may be difficult to manage or require extra work hours or cost to do so.

4-4. MPPP With Fuzzy Costs and Fuzzy Demand

Now we will consider a simple two-period production planning problem as shown in Table 4-7. In this example, all costs and demands are given by interval estimates.

(1) Solution by Crisp Dynamic Programming

The translated scalar costs and demands are given in

TABLE 4-7.

t	D_t (unit)	A_t (\$1,000)	c_t (\$1,000)	h_t (\$1,000)
1	(20,30,60)	(20,30,40)	(1,4,6)	(1,2,3)
2	(30,50,60)	(20,40,50)	(2,3,5)	(1,3,4)

Table 4-8.

t	D_t	A_t	c_t	h_t
1	30	30	4	2
2	50	40	3	3

table 4-8.

Solution:

$$Z_1 = C_{01} = A_1 + c_1(D_1) = 30 + 4(30) = 150$$

$$Z_1^* = 150$$

$$Z_2 = \begin{cases} C_{02} = A_1 + c_1(D_1 + D_2) + h_1(D_2) \\ \quad = 30 + 4(30+50) + 2(50) = 450 \\ Z_1^* + C_{12} = Z_1^* + A_2 + c_2(D_2) \\ \quad = 150 + 40 + 3(50) = 340 \end{cases}$$

The optimal schedule is shown in Table 4-9.

Table 4-9.

Period	1	2
Production	30	50

(2) Solution by Fuzzy Dynamic Programming

$$Z_1^* = C_{01} = A_1 (+) c_1 (.) D_1$$

$$= (20, 30, 40) + (1, 4, 6) (.) (20, 30, 60)$$

$$= \begin{cases} 0 & x \leq 40 \\ [-80 + (1600 + 120x)^{1/2}] / 60 & 40 < x \leq 150 \\ [310 - (100 + 240x)^{1/2}] / 120 & 150 < x \leq 400 \\ 0 & x > 400 \end{cases}$$

$$\begin{aligned} Z_0 (+) C_{02} &= A_1 (+) c_1 (.) [D_1 (+) D_2] (+) h_1 (.) [D_1 (+) D_2 (-) D_1] \\ &= (20, 30, 40) (+) (1, 4, 6) (.) [(20, 30, 60) (+) \\ &\quad (30, 50, 60)] (+) (1, 2, 3) (.) [(20, 30, 60) \\ &\quad (+) (30, 50, 60) (-) (20, 30, 60)] \end{aligned}$$

$$= \begin{cases} 0, & x \leq 60 \\ [-240 + (21600 + 600x)^{1/2}] / 300 & 60 < x \leq 450 \\ [740 - (-3600 + 520x)^{1/2}] / 260 & 450 < x \leq 1060 \\ 0 & x > 1060 \end{cases}$$

$$\begin{aligned} Z_2 \left\{ \begin{aligned} Z_1^* (+) C_{12} &= Z_1^* (+) A_2 (+) c_2 (.) D_2 \\ &= Z_1^* (+) (20, 40, 50) (+) (2, 3, 5) (.) (30, 50, 60) \end{aligned} \right. \end{aligned}$$

$$= \begin{cases} 0 & x \leq 120 \\ [-170 + (4900 + 200x)^{1/2}] / 100 & 120 < x \leq 340 \\ [490 - (100 + 320x)^{1/2}] / 160 & 340 < x \leq 750 \\ 0 & x > 750 \end{cases}$$

Using Lee-Li's method,

$$M_u[Z_1^*(+)C_{12}] < M_u[Z_0(+)C_{02}]$$

so $Z_2^* = [Z_1^*(+)C_{12}]$ and the optimal production schedule is:

Table 4-10.

period	1	2
Production	$D_1 = (20, 30, 60)$	$D_2 = (30, 50, 60)$

(3) Comparison of the solutions

The solutions obtained by the crisp model and by the fuzzy model are completely different. The former is represented by single numbers and has lost much of the original information. The later is represented by fuzzy numbers and still retains the interval nature. As discussed in sections 2 and 3 of this chapter, this difference is very important and useful in practice.

The results of our examples indicate that we have found an effective method--the general fuzzy approach by the use of fuzzy numbers to solve the fuzzy MPPP problems. The above examples are simple and are suited to be

solved by the W-W approach. This is for the simplification of their computation. These examples are used only to demonstrate the procedures and advantages of the general fuzzy approach by the use of fuzzy numbers. Many real-world MPPP problems with different constraints can be solved by the combination of the general fuzzy approach and computer computation.

4-5. Comparison of Three Fuzzy MPPP Methods

Three fuzzy MPPP methods have been proposed. They are: Sommer's model, Kacprzyk-Staniewski model, and the method developed in this work. In this section, we will compare these three methods by discussing their related examples from the practical point of view.

(1) Sommer's Example

Sommer [63] solved a numerical example of a fuzzy MPPP problem to demonstrate his method. This problem was discussed in Chapter 2, and the B-Z approach was used.

Mathematically, his problem is described as:

Let the membership function of production level be:

$$\mu_C(P_i) = \begin{cases} 0 & \text{if } 0 \leq P_i \leq 60-10i \\ -3 + .5i + P_i/20 & \text{if } 60-10i < P_i \leq 80-10i \\ 5 - .5i - P_i/20 & \text{if } 80-10i \leq P_i \leq 100-10i \\ 0 & \text{if } 95-10i < P_i \end{cases} \quad (1)$$

and the membership function of the inventory level at the end of the planning horizon be:

$$(x_{N+1}) = \begin{cases} 1-x_{N+1}/20 & \text{if } 0 \leq x_{N+1} \leq 20 \\ 0 & \text{elsewhere} \end{cases} \quad (2)$$

The planning horizon is $N = 4$, the stock level at the beginning is $x_1 = 0$, and the demands are $D_1=45$, $D_2=50$, $D_3=45$, $D_4=60$.

In this example, both demand and inventory are crisp numbers, but the two goals--"production level decrease as steadily as possible" and "the ending inventory level as low as possible"--are fuzzy. It seems that the fuzzy environment of this production planning problem is provided artificially rather than naturally. The reason we employ a fuzzy set or a fuzzy number to indicate a goal is that the goal cannot be precisely indicated or determined because precise information is lacking. However, when exact information is available, such as demand and inventory in the example above, the decision maker should not set up a fuzzy goal. This is because the goal set up (without calculation) by the decision maker is often either suboptimal or infeasible.

In Sommer's example, if we think of the goal "the ending inventory level shall be as low as possible" as a constraint $x_{N+1}=0$ and his other goal "the production level shall decrease as steadily as possible" as a function of

time t , $dp/dt=f(t)$, then we can easily get the optimal production schedule by crisp dynamic programming techniques.

(2) Kacprzyk-Staniewski's (K-S's) Example [67]

The conventional approaches to production and inventory control usually involve optimization of a performance function consisting of some cost-related terms. Kacprzyk and Staniewski transformed the cost optimization problem into one of maintaining some desired inventory level. That is, they believe the average costs usually are known to be some functions of inventory and they are mostly isomorphic. The same applies to replenishment or production.

In their example, it is assumed that $X = \{1, \dots, 10\}$;
 $U, Y = \{1, \dots, 5\}$.

Given:

the reference fuzzy inventory level S_1, \dots, S_5 ;

the reference fuzzy replenishment C_1, \dots, C_3 ;

the fuzzy demand D and fuzzy goal G ;

the fuzzy constraints $C(S_1), \dots, C(S_5)$.

Obviously, the difference between this problem and Sommer's problem is that both its demand and inventory are fuzzy. This problem is more fuzzy and more reasonable than Sommer's problem in the sense of fuzzy sets theory. But from the practical point of view, it is too hard for

the decision maker to provide the necessary and consistent information for determining the membership function of the S_i 's, C_j 's and $C(S_i)$'s. This is especially true where i and j are big numbers.

As an alternative to using Kacprzyk and Staniewski's method, the problem can be solved by a procedure similar to our general fuzzy approach.

Namely:

(a) use an appropriate fuzzy number, I^* , to indicate the "desired inventory level";

(b) use the transaction function

$$I^* = I_{t-1} (+) P_t (-) D_t$$

and the given I_0 and D_t to determine P_t , $t=1, \dots, N$, where P_t is the fuzzy replenishment or production to maintain the desired inventory level and satisfy the fuzzy demand;

(c) consider both P_t and the constraints on the replenishment or production simultaneously to determine the optimal replenishment policy;

(d) if the cost functions related to inventory level, $f(I_t)$ and replenishment level, $f(R_t)$, are known, we can use the total cost function $F = f(I_t) (+) f(R_t)$ to determine the optimal policy.

Following the above procedure, all the decision maker has to do is provide some estimates: the desired

inventory level, the replenishment constraints, and, if possible, the cost functions.

(3) Our Examples

The fuzzy environment provided in our three examples is natural. We can conveniently get the information of fuzzy costs and fuzzy demands by asking the decision maker to provide the related estimates. Our method not only gives the optimal fuzzy schedule but also the total minimum fuzzy cost. From a practical view point, these are very important and useful.

Thus, we can summarize the essential features of the three fuzzy MPPP methods in the following table.

TABLE 4-11

Comparison of Three Fuzzy MPPP Methods

	Sommer's (1981)	K-S's (1981)	Ours (1989)
Goal	Fuzzy	Fuzzy	Fuzzy
Constraint	Fuzzy	Fuzzy	Fuzzy
Demand	Crisp	Fuzzy	Fuzzy
Inventory	Crisp	Fuzzy	Fuzzy
Costs	-----	-----	Fuzzy
Other Conditions	Same	Same	Same
Fuzzy Environment	Artificial	Natural	Natural
Transition Equation	$I_{t+1} = I_t + P_t - D_t$	$I_{t+1} = I_t (+) P_t (-) D_t$	$I_{t+1} = I_t (+) P_t (-) D_t$
Optimization Function	Max. $(\mu_G \wedge \mu_C)$	Max. $(\mu_G \wedge \mu_C)$	Min. (Total Cost)
Schedule	Most Satisfaction of Decision Maker	Most Satisfaction of Decision Maker	Minimization of Total Cost of Company

Table 4-11 (Continued).

Data Collection	Medium	Difficulty	Easy
computation	Simple	Tedious	Medium
Fuzzy Multi-stage Decision Processes	Bellman & Zadeh's Model	Bellman & Zadeh's Model	General Fuzzy Approach by The Use of Fuzzy Numbers

CHAPTER 5. OTHER APPLICATIONS OF THE GENERAL FUZZY APPROACH BY THE USE OF FUZZY NUMBERS

5-1. Introduction

The procedures and examples of Chapter 4 indicate that the general fuzzy approach by the use of fuzzy numbers is an effective technique. Thus, one would think that there are many other problems, such as fuzzy resource allocation problems, which should be solved by the same procedure in order to obtain more reasonable solutions. In this chapter, we will use the general fuzzy approach to solve three other kinds of fuzzy multi-stage decision making problems.

5-2. The Network Problem

A man who lives in city A is going to visit city B.

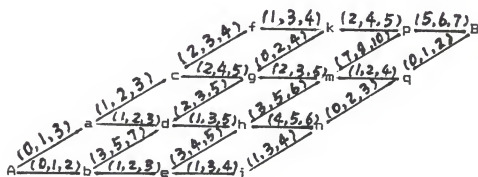


Figure 5-1. The Network Problem

His possible routes and corresponding estimated costs (in hundred of dollars) are shown in Figure 5-1. Please help him to determine the best route in the view of minimizing the total traveling cost.

Solution:

St- age j	St- ate j	Con- trol j+1	Cost of j+1	Pre- State	Total Cost	Best Route
6	p	d	(5,6,7)	B	(5, 6, 7)	d
	q	u	(0,1,2)	B	(0, 1, 2)	u
5	k	d	(2,4,5)	p	(2,4,5) (+) (5,6,7) = (7,10,12)	d
	m	u	(7,9,10)	p	(7,9,10) (+) (5,6,7) = (14,15,17)	
		d	(1,2,4)	q	(1,2,4) (+) (0,1,2) = (1,3,6)*	d
	n	u	(0,2,3)	q	(0,2,3) (+) (0,1,2) = (0,3,5)	u
4	f	d	(1,3,4)	k	(1,3,4) (+) (7,10,12) = (8,13,16)	d
	g	u	(0,2,4)	k	(0,2,4) (+) (7,10,12) = (7,12,16)	
		d	(2,3,5)	m	(2,3,5) (+) (1,3,6) = (3,6,11)*	d
	h	u	(3,5,6)	m	(3,5,6) (+) (1,3,6) = (4,8,12)	
		d	(4,5,6)	n	(4,5,6) (+) (0,3,5) = (4,8,11)*	d
	j	u	(1,3,4)	n	(1,3,4) (+) (0,3,5) = (1,6,9)	u
3	c	u	(2,3,4)	f	(2,3,4) (+) (8,13,16) = (10,16,20)	
		d	(2,4,5)	g	(2,4,5) (+) (3,6,11) = (5,10,16)*	d
	d	u	(2,3,5)	g	(2,3,5) (+) (3,6,11) = (5,9,16)*	u

	d	(1, 3, 5)	h	(1, 3, 5) (+) (4, 8, 11) = (5, 11, 16)	
	e	u	(3, 4, 6)	h	(3, 4, 5) (+) (4, 8, 11) = (7, 12, 16)
	d	(1, 3, 4)	j	(1, 3, 4) (+) (1, 6, 9) = (2, 9, 13)*	d
<hr/>					
2	a	u	(1, 2, 3)	c	(1, 2, 3) (+) (5, 10, 16) = (6, 12, 19)
	d	(1, 2, 3)	d	(1, 2, 3) (+) (5, 9, 16) = (6, 11, 19)*	d
	b	u	(3, 5, 7)	d	(3, 5, 7) (+) (5, 9, 16) = (8, 14, 23)
	d	(1, 2, 3)	e	(1, 2, 3) (+) (2, 9, 13) = (3, 11, 16)*	d
<hr/>					
1	A	u	(0, 1, 3)	a	(0, 1, 3) (+) (6, 11, 19) = (6, 12, 22)
	d	(0, 1, 2)	b	(0, 1, 2) (+) (3, 11, 16) = (3, 12, 18)*	d
<hr/>					

The minimum total cost from A to B is (3, 12, 18). The best route is:

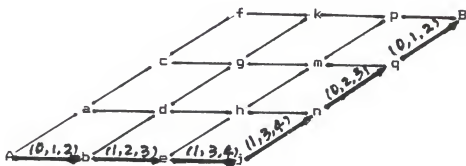


Figure 5-2. The Best Route

From the above solution, the man know not only his best route, but also that his total cost is in the interval of 3 to 18 hundred dollars.

If we translate the above estimated costs into scalars and solve this problem by dynamic programming, we will find two best routes (Fig.5-3) and the minimum total cost of 12 hundred dollars.

By comparison, we know:

(1) the route A-a-d-g-m-q-B's corresponding total cost is (6,12,22), which is more than the route A-b-e-j-n-q-B's total cost (3,12,18). Route A-b-e-j-n-q-B is the real optimal route. Route A-a-d-g-m-q-B is a suboptimal route.

(2) the crisp total cost of 12 hundred dollars has lost a considerable amount of useful information. If the man brings only 12 hundred dollars to travel, he might get into trouble because of using all his money before arriving at city B.

Obviously, the solution of the fuzzy method is better than the solution of the crisp method in the practical point of view.

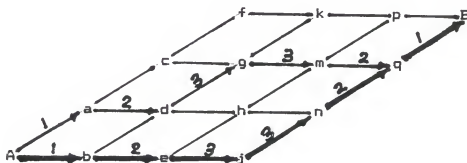


Figure 5-3. The Two Best Route Obtained by Crisp Method

5-3. Fuzzy Resource Allocation Problem

Table 5-1 displays a fuzzy resource allocation problem. The second column lists the different proposals. The third column is the amount of money (in millions of dollars) allocated for this particular proposal and the last column shows the amount that the profit increases (in

TABLE 5-1

Cosmetic (Stage)	Proposal (k)	Money Allocated (u(k))	Profit Increases (R)
1	1	0	0
	2	1	(1, 5, 6)
	3	2	(6, 7, 11)
	4	3	(6, 8, 10)
	5	4	(7, 9, 11)
2	1	0	0
	2	2	(7, 8, 9)
	3	4	(10, 12, 14)
3	1	0	0
	2	1	(0, 3, 4)
	3	3	(4, 5, 6)
	4	4	(9, 11, 13)

million of dollars) due to this allocation.

It should be noted that the numbers in the last column of the table are fuzzy numbers. This is because the profit increase due to each allocation cannot be precisely determined at the time of decision making.

The total resource available is \$4 million. Our problem is to determine how to allocate this money so that a maximum increase in profit can be obtained.

SOLUTION:

STAGE 3

X_2	K_3	R_3	$f_3(X_2)$	MAXIMUM	$f_3(X_2)$	K_3
0	1	0	0	0		1
1	1	0	0	0		
	2	(0, 3, 4)	(0, 3, 4)	(0, 3, 4)		2
2	1	0	0	0		
	2	(0, 3, 4)	(0, 3, 4)	(0, 3, 4)		2
3	1	0	0			
	2	(0, 3, 4)	(0, 3, 4)			
	3	(4, 5, 6)	(4, 5, 6)	(4, 5, 6)		3
4	1	0	0			
	2	(0, 3, 4)	(0, 3, 4)			
	3	(4, 5, 6)	(4, 5, 6)			
	4	(9, 11, 13)	(9, 11, 13)	(9, 11, 13)		4

STAGE 2

X_1	K_2	R_2	$f_2(X_1) = R_2 + f_1(X_1 - u(K_2))$	$\text{Max } f_2(X_1)$	K_2
0	1	0	0	0	1
1	1	0	(+) (0, 3, 4)	(0, 3, 4)	1
2	1	0	(+) (4, 5, 6)		
	2	(7, 8, 9)	(+) 0	(7, 8, 9)	2
3	1	0	(+) (4, 5, 6)		
	2	(7, 8, 9) (+) (0, 3, 4) = (7, 11, 13)		(7, 11, 13)	2
4	1	0	(+) (9, 11, 13) = (9, 11, 13)		
	2	(7, 8, 9) (+) (0, 3, 4) = (7, 11, 13)			
	3	(10, 12, 14) (+) 0 = (10, 12, 14)		(10, 12, 14)	3

STAGE 1

X_0	K_1	R_1	$f_1(X_0) = R_1 + f_2(X_0 - u(K_1))$	$\text{Max } f_1(X_0)$	K_1
4	1	0	(+) (10, 12, 14) = (10, 12, 14)		
	2	(1, 5, 6)	+ (7, 11, 13) = (7, 11, 13)		
	3	(6, 7, 11) + (7, 8, 9) = (13, 15, 20)		(13, 15, 20)	3
	4	(6, 8, 10)	+ (0, 3, 4) = (6, 11, 14)		
	5	(8, 9, 10)	+ 0 = (8, 9, 10)		

The optimal solution can now be obtained directly from the tables above. Starting from Stage 1, the maximum

return with \$4 million to allocate to stage 1, 2 and 3 is:

$$f_0(4) = (13, 15, 20)$$

The optimal proposal for stage 1 is proposal 3 in which \$2 million allocated, for stage 2 is proposal 2 in which \$2 million allocated and for stage 3 is proposal 1 in which \$0 million is allocated (Table 5-2).

Table 5-2.

STAGE	1	2	3
PROPOSAL	3	2	1
MONEY ALLOCATED	2	2	0

Here again, if we translate the fuzzy numbers into single numbers as shown in Table 5-3 and solve this problem by crisp dynamic programming, the optimal proposals are as shown in Table 5-4.

If the firm employs the proposals showed in table 5-4, the total maximum profit increase will be \$(8, 16, 19), which is less than \$(13, 15, 20) (see Fig.5-4). In addition, the management of the firm will know that the maximum profit is an interval amount between \$13 to \$20 million by the fuzzy solution.

TABLE 5-3.

Cosmetic (Stage)	Proposal (k)	Money Allocated (u(k))	Profit Increases (R)
1	1	0	0
	2	1	5
	3	2	7
	4	3	8
	5	4	9
2	1	0	0
	2	2	8
	3	4	12
3	1	0	0
	2	1	3
	3	3	5
	4	4	11

Table 5-4.

STAGE	1	2	3
PROPOSAL	2	2	2
MONEY ALLOCATED	1	2	1

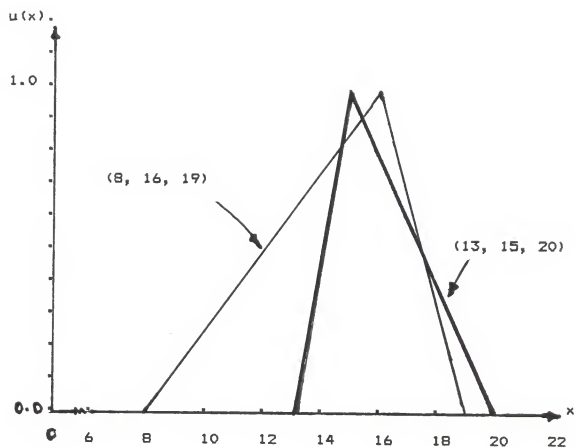


Figure 5-4. Comparison of fuzzy profits

4-3. Taxi Cab Driver's Best Strategy

--An Example of Fuzzy Stochastic Processes

PROBLEM:

A taxi cab driver picks up and delivers passengers in only three cities of M, J and A. He can pick up his passengers by cruising around, staying at cab stations and/or listening to the radio call. His following transition matrix is obtained by statistical analysis of the past data. His profit matrix is obtained by experienced estimations.

Cruise

$$P^C = \begin{matrix} & \begin{matrix} M & J & A \end{matrix} \\ \begin{matrix} M \\ J \\ A \end{matrix} & \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} \end{matrix} \quad R^C = \begin{matrix} & \begin{matrix} M & J & A \end{matrix} \\ \begin{matrix} M \\ J \\ A \end{matrix} & \begin{bmatrix} (9,10,11) & (3,4,5) & (7,8,9) \\ (12,14,16) & 0 & (16,18,20) \\ (9,10,11) & (1,2,3) & (8,10,12) \end{bmatrix} \end{matrix}$$

Cab Station

$$P^{CS} = \begin{matrix} & \begin{matrix} M & J & A \end{matrix} \\ \begin{matrix} M \\ J \\ A \end{matrix} & \begin{bmatrix} 1/16 & 3/4 & 3/16 \\ 1/16 & 7/8 & 1/16 \\ 1/8 & 3/4 & 1/8 \end{bmatrix} \end{matrix} \quad R^{CS} = \begin{matrix} & \begin{matrix} M & J & A \end{matrix} \\ \begin{matrix} M \\ J \\ A \end{matrix} & \begin{bmatrix} (7,8,9) & (1,2,3) & (0,4,8) \\ (4,8,12) & (12,16,20) & (4,8,12) \\ (5,6,7) & (1,4,7) & (1,2,3) \end{bmatrix} \end{matrix}$$

Radio Call

$$P^{RC} = \begin{matrix} & \begin{matrix} M & J & A \end{matrix} \\ \begin{matrix} M \\ J \\ A \end{matrix} & \begin{bmatrix} 1/4 & 1/8 & 5/8 \\ \text{---} & \text{---} & \text{---} \\ 3/4 & 1/16 & 3/16 \end{bmatrix} \end{matrix} \quad R^{RC} = \begin{matrix} & \begin{matrix} M & J & A \end{matrix} \\ \begin{matrix} M \\ J \\ A \end{matrix} & \begin{bmatrix} (1,4,7) & (4,6,8) & (2,4,6) \\ \text{---} & \text{---} & \text{---} \\ (3,4,5) & 0 & (4,8,12) \end{bmatrix} \end{matrix}$$

Suppose the cab driver starts in city M in the morning, obtain the best alternative to maximize his profit for the day if he plans to make only 3 deliveries during the day.

Solution:

First we calculate the expected profits V_i^k with $i=(M, J, A)$ and $k=(c, cs, rc)$. Then we find his best strategy by the general fuzzy approach.

$$\begin{aligned} V_M^c &= 1/2(.) (9, 10, 11) (+) 1/4(.) (3, 4, 5) (+) 1/4(.) (7, 8, 9) \\ &= (7, 8, 9) \end{aligned}$$

$$\begin{aligned} V_J^c &= 1/2(.) (12, 14, 16) (+) 1/2(.) (16, 18, 20) \\ &= (14, 16, 18) \end{aligned}$$

$$\begin{aligned} V_A^c &= 1/4(.) (9, 10, 11) (+) 1/4(.) (1, 2, 3) (+) 1/2(.) (8, 10, 12) \\ &= (6.5, 8, 9.5) \end{aligned}$$

$$\begin{aligned} V_M^{cs} &= 1/16(.) (4, 8, 12) (+) 3/4(.) (1, 2, 3) (+) 3/16(.) (0, 4, 8) \\ &= (1, 2.75, 4.5) \end{aligned}$$

$$\begin{aligned} V_J^{cs} &= 1/16(.) (4, 8, 12) (+) 7/8(.) (12, 16, 20) (+) 1/16(.) (4, 8, 12) \\ &= (11, 15, 19) \end{aligned}$$

$$\begin{aligned} V_A^{cs} &= 1/8(.) (5, 6, 7) (+) 3/4(.) (1, 4, 7) (+) 1/8(.) (1, 2, 3) \\ &= (1.5, 4, 6.5) \end{aligned}$$

$$\begin{aligned} V_M^{rc} &= 1/4(.) (1, 4, 7) (+) 1/8(.) (4, 6, 8) (+) 5/8(.) (2, 4, 6) \\ &= (2, 4.25, 6.5) \end{aligned}$$

$$V_J^{rc} = --$$

$$\begin{aligned} V_A^{rc} &= 3/4(.) (3, 4, 5) (+) 1/16(.) (0) (+) 3/16(.) (4, 8, 12) \\ &= (3, 4.5, 6) \end{aligned}$$

$$v_i^k$$

i	k	c	cs	rc
M		(7, 8, 9)	(1, 2.75, 4.5)	(2, 4.25, 6.5)
J		(14, 16, 18)	(11, 15, 19)	-----
A		(6.5, 8, 9.5)	(1.5, 4, 6.5)	(3, 4.5, 6)

STAGE 3.

City Alter- (Stage j) (+) (Stage j+1) = Total Return (Best
natives) Return Return Policy)

M	c	(7, 8, 9) (+)	0	(7, 8, 9)*	c
	cs	(1, 2.75, 4.5) (+)	0	(1, 2.75, 4.5)	
	rc	(2, 4.25, 6.5) (+)	0	(2, 4.25, 6.5)	
J	c	(14, 16, 18) (+)	0	= (14, 16, 18)*	c
	cs	(11, 15, 19) (+)	0	= (11, 15, 19)	
	rc	-----		-----	
A	c	(6.5, 8, 9.5) (+)	0	= (6.5, 8, 9.5)*	c
	cs	(1.5, 4, 6.5) (+)	0	= (1.5, 4, 6.5)	
	rc	(3, 4.5, 6) (+)	0	= (3, 4.5, 6)	

STAGE 2.

City Alter-	(Stage j (+)	(Stage j+1 = Total Return (Best
natives)	Return)	Return) Policy)

M c (7,8,9) (+) 1/2(.) (7,8,9) (+) 1/4(.) (14,16,18)
 (+) 1/4(.) (6.5,8,9.5) = (15.625, 18, 20.375)* c
 cs (1,2.75,4.5) (+) 1/16(7,8,9) (+) 3/4(.) (14,16,18)
 (+) 3/16(.) (6.5,8,9.5) = (13.06, 16.75, 20.34)
 rc (2,4.25,6.5) (+) 1/4(.) (7,8,9) (+) 1/8(.) (14,16,18)
 (+) 5/8(.) (6.5,8,9.5) = (9.56, 13.25, 16.94)

J c (14,16,18) (+) 1/2(.) (7,8,9) (+) 0 (+) 1/2(.) (6.5,8,9.5)
 = (20.75, 24, 27.25)
 cs (11,15,19) (+) 1/16(.) (7,8,9) (+) 7/8(.) (14,16,18)
 (+) 1/16(.) (6.5,8,9.5) = (24.09, 30, 35.91)* cs
 rc -----

A c (6.5,8,9.5) (+) 1/4(.) (7,8,9) (+) 1/4(.) (14,16,18)
 (+) 1/2(.) (6.5,8,9.5) = (15, 18, 21)* c
 cs (1.5,4,6.5) (+) 1/8(.) (7,8,9) (+) 3/4(.) (14,16,18)
 (+) 1/8(.) (6.5,8,9.5) = (13.69, 18, 22.31)
 rc (3,4.5,6) (+) 3/4(.) (7,8,9) (+) 1/16(.) (14,16,18)
 (+) 3/16(.) (6.5,8,9.5) = (7.34, 8.5, 9.66)

STAGE 1

City	Alter-	Stage j (+)	Stage j+1 = Total Return	Best
natives	Return	Return		Policy

M c (7,8,9) (+) 1/2(.) (15.625, 18, 20.375) (+) 1/4
(.) (24.09, 30, 35.91)

(+) 1/4(.) (15, 18, 21) = (24.58, 29, 33.42)

cs (1, 2.75, 4.5) (+) 1/16(.) (15.625, 18, 20.375)
(+) 3/4(.) (24.09, 30, 35.91) (+) 3/16(.) (15, 18, 21)
= (22.85, 29.75, 36.65)*

rc (2, 4.25, 6.5) (+) 1/4(.) (15.625, 18, 20.375)
(+) 1/8(.) (24.09, 30, 35.91) (+) 5/8(.) (15.18, 21)
= (18.31, 23.75, 29.215)

The expected maximum total return is \$(22.85, 29.75, 36.65). The cab driver's best strategy is shown in Figure 5-4.

1st passenger	2nd passenger	3rd passenger
City Alternatives	City Alternatives	City Alternatives

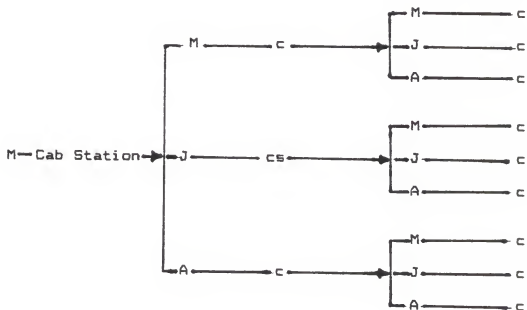


Figure 5-4. The Cab Driver's Best Strategy

CHAPTER 6. CONCLUSIONS AND SUGGESTIONS

The purpose of this study was to find an effective method to solve the interval MPPP problem and to retain all the original information.

As shown, this purpose was reached via the following general fuzzy approach by the use of fuzzy numbers:

- (1) employing appropriate fuzzy numbers to represent the interval estimates in the multi-stage decision problems;
- (2) using the operations of fuzzy numbers combined with dynamic programming to solve the problem;
- (3) determining the required minimum or required maximum fuzzy number by the use of fuzzy number ranking techniques.

Three MPPP example problems with fuzzy costs and/or fuzzy demands were solved by this general fuzzy approach. The solutions, both production schedule and/or minimum total cost, were indicated by fuzzy numbers. This kind of solution can give the management clear ideas about the interval units of production at each period and the interval amounts of the required minimum total cost. Therefore, the management can manage the production flexibly and control the cost confidently. The crisp model, by comparison, indicated solutions by single numbers and lost a considerable amount of useful information. All the management can do is to produce the

exact number of products shown in the schedule and expect to spend the so called "minimum total coat". The fuzzy representation of schedule and minimum total cost is clearly more practical.

The second advantage of the general fuzzy approach is that it breaks the same-space restriction. It can be used to solve not only the problem with fuzzy goals and fuzzy constraints defined in the same space, but also the problem with fuzzy goals and constraints defined in different spaces. This advantage, in our view, will result in wide application of dynamic programming to fuzzy systems. The three other examples of fuzzy multi-stage decision making problems given in Chapter 5 support this view.

In addition, the general fuzzy approach by the use of fuzzy numbers partially overcomes the difficulty of the computational requirement for large dimensional problems. The information needed to solve the real-world problems is easy to collect, and the computations may be carried out by computer.

As areas for further study, we suggest (1) use the general fuzzy approach by the use of fuzzy numbers to solve real-world MPPP problems by computer and (2) apply the general fuzzy approach to various fuzzy multi-stage decision making problems.

REFERENCES

1. Barancsi, E, et al, " A report of research on inventory models", Engineering Costs and Production Economics, 7, pp127-136, 1983.
2. Bowman, Edward, H., " Production scheduling by the transportation method of linear programming," Operations Research, 3, 1956.
3. Buffa, Elwood and Miller, J.G., Production Inventory Systems: Planning and Control, Richard D. Irwin, 1979.
4. Hanssman, Fred, and Hess, S.W., " A linear programming approach to production and employment scheduling," Management Technology, January, 1960.
5. Hax, Arnold C., " Aggregate production planning," Handbook of Operations Research: Models and Applications, Salah E. Elmagraby, and Joseph J. Moder, (eds.) New York, N. Y.: Van Nostrand Reinhold Co., 1978.
6. Holt, Charles C., J Franco Modigliani, John F. Muth, and Herbert A. Simon, Planning Production, Inventories, and Workforce. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1960.

7. Bellman, R. E., Dynamic Programming, Princeton Univ. Press, Princeton, N. J., 1957.
8. Elsayed, Elsayed A., Analysis and Control of Production Systems, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1985.
9. Axsater, S., Schneeweiss, Ch. and Silver, E.(eds.), MultiStage Production Planning and Inventory Control, Springer-Verlag Berlin Heidelberg, 1986.
10. Wanger, Harvey M. and Whitin, Thomas M., "A dynamic version of the economic lot size model", Management Science, 5, 1958.
11. Wagner, Harvey M., "A postscript to 'Dynamic problems of the firm'", Naval Research Logistics Quarterly, 7, 1960.
12. Zangwill, Willard I., "A deterministic multiperiod production scheduling model with backlogging", Management Science, 13, 1966.
13. Dixon, P. S., and Silver, E. A.: " A heuristic solution procedure for the multi-item single level limited capacity lot sizing problem," Journal of Operations Management, 2, 23-40, 1981.

14. Van Nunen, J. A., and Wessels, J.: " Multi-item lot size determination and scheduling under capacity constraints," European Journal of Operational Research, 2, 36-41, 1978.
15. Aras, O.A., and Swanson, L.A.: " A lot sizing and sequencing algorithm of dynamic demands upon a single facility," Journal of Operations Management, 2, 177-186, 1982
16. Williams, W. William., Peters, Michael H., and Raiszadeh, M.E.,: "Time-dependent demand in requirements planning: An exploratory assessment of the effects of serially correlated demand sequences on lot-sizing performance," Journal of Operations Management, 6, 69-86, 1985.
17. Orlicky, J. A., Material Requirements Planning. NY: McGraw-Hill, Inc., 1975.
18. Wemmerlov, U. " The ubiquitous EOQ-Its relation to discrete lot-sizing heuristics," International Journal of Operations and Production Management, 20, 15-35, 1979.
19. Wemmerlov, U. "The part-period balancing algorithm and its look ahead-look back feature: A theoretical and experimental analysis of a single stage lot-sizing

procedure." Journal of Operations Management, 4, 23-39, 1983.

20. Silver, E.A., and H.C. Meal. "A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment." Production and Inventory Management, 14, 64-74, 1973.

21. James H. Bookbinder and Bak-Tee H'ng, "Production lot sizing for deterministic rolling schedules," Journal of Operations Management, 6, 349-362, 1986.

22. Karni, R. "Maximum part-period gain (MPG)-A lot sizing procedure for unconstrained and constrained requirements planning systems." Production and Inventory Management, 22, 91-98, 1981.

23. Bookbinder, J.H. and J.Y. Tan. "Two lot-sizing heuristics for the case of deterministic time-varying demands." International Journal of Operations and Production Management, 5, 30-42, 1985.

24. Silver, E.A. and J. Miltenburg. "Two modifications of the Silver-Meal lot-sizing heuristic." INFOR, 22, 56-69, 1984.

25. Urban Wemmerlov, "The behavior of lot-sizing

procedures in the presence of forecast errors." Journal of Operations Management, 8, 37-47, 1989.

26. Silver, E.A. "The response of a concerned management scientist." Production and Inventory Management, 17, 96-99, 1976.

27. McLaren, B.J. " A study of multiple level lot sizing procedures for material requirements planning systems." Ph.D dissertation, School of Management, Purdue University, May 1977.

28. Groff, G.K. " A lot sizing rule for time-phased component demand." Production and Inventory Management, 20, 47-53, 1979.

29. Paul C. Jordan, " A comparative analysis of the relative effectiveness of four dynamic lot-sizing techniques on return on investment," Decision Science, 20, 134-141, 1989.

30. William W. Trigeiro, L. Joseph Thomas, and John O. McClain, "Capacitated lot sizing with setup times," Management Science, 35, 353-367, 1989.

31. Klaus Zoller and Andreas Robrade, " Dynamic lot sizing techniques: Survey and comparison," Journal of Operations Management, 7, 125-145, 1989.

32. T.L. Morin, "Computational advances in dynamic programming," in: M.L. Puterman, (eds.), Dynamic Programming and Its Applications, Academic Press, New York, 53-90, 1978.

33. T.L. Morin and A.O. Esogbue, "Imbedded state approach to reduction of dimensionality in dynamic programming of high dimensions," Journal of Mathematical Analysis and Applications, 48, 1974.

34. T.L. Morin and A.O. Esogbue, "Some efficient dynamic programming algorithms for the optimal sequencing and scheduling of water supply projects," Water Resources Research, 7, 1971.

35. C.A. Shoemaker, "Applications of dynamic programming and other optimization methods in pest management," IEEE Transactions on Automatic Control, 26, 1981.

36. Efstathiou, J., "Practical multi-attribute decision-making and fuzzy set theory," in: Zimmermann et al., 307-322, 1984.

37. Freeling, A. N. S., "Fuzzy sets and decision analysis," IEEE Trans. SMC 10, 341-354, 1980.

38. Tong, R.M., Bonissone, P. P., "Linguistic solutions to fuzzy decision problems," in: Zimmermann et al., 323-334,

1984.

39. Yager, R.R., "Fuzzy decision making including unequal objectives," FSS 1, 87-95, 1978.

40. Zimmermann, H. J., Zadeh, L. A., Gaines, B.R. (eds.). Fuzzy Sets and Decision Analysis, Amsterdam, New York, Oxford, 1984.

41. Borisov, A., Krumberg, O., "A theory of possibility for decision making," FSS9, 13-23, 1983.

42. V. Rajkovic, J. Efstathiou and M. Bohanec, "A concept of rule-based decision support systems," in: J. Kacprzyk and S.A. Orlovski (eds.), Optimization Models Using Fuzzy Sets and Possibility Theory, D. Reidel Publishing Company, Holland, 1987.

43. Th. Whalen, "Introduction to decision making under various kinds of uncertainty," in: J. Kacprzyk and S.A. Orlovski (eds.), Optimization Models Using Fuzzy Sets and Possibility Theory, D. Reidel Publishing Company, Holland, 1987.

44. V.I. Glushkov and A.N. Borisov, "Analysis of fuzzy evidence in decision making models" in: J. Kacprzyk and S.A. Orlovski (eds.), Optimization Models Using Fuzzy Sets and Possibility Theory, D. Reidel Publishing Company,

Holland, 1987.

45. V.E. Zhukovin, "A fuzzy multicriteria decision making model," in: J. Kacprzyk and S.A. Orlovski (eds.), Optimization Models Using Fuzzy Sets and Possibility Theory, D. Reidel Publishing Company, Holland, 1987.

46. J.J. Buckley, "Fuzzy programming and the multicriteria decision problem," in: J. Kacprzyk and S.A. Orlovski (eds.), Optimization Models Using Fuzzy Sets and Possibility Theory, D. Reidel Publishing Company, Holland, 1987.

47.. Adamo, J.M.: "Fuzzy Decision Trees," FSS4, 207-219, 1980.

48. Asai, K., Tanaka, H., Okuda, T., "Decision-making and its goal in a fuzzy environment," in: Zadeh et al. (eds.) 257-277, 1975.

49. Chang, S.S.L., "On risk and decision making in a fuzzy environment," in: Zadeh et al. (eds.) 219-226, 1975.

50. Cheng, Y.Y.M., McInnis, B., "An algorithm for multiple attribute, multiple alternative decision problems based on fuzzy sets with application to medical diagnosis, IEEE Trans. SMC 10, 645-650, 1980.

51. Dompere, K.K., " The theory of fuzzy decisions," in: Gupta, Sanchez (eds.) 365-380, 1982.
52. Kacprzyk, J., Yager, R.R., (eds.) Management Decision Support Systems Using Fuzzy Sets and Possibility Theory, Koln, 1985.
53. Nurmi, H., " Approaches to collective decision making with fuzzy preference relations," FSS 6, 249-259, 1981.
54. R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment", Management Science 17, B141-B164, 1970.
55. S.S.L. Chang, " Fuzzy dynamic programming and the decision making process," in: Proc. 3rd Princeton Conference on Information Sciences and Systems, 200-203, 1969.
56. B. Gluss, " Fuzzy multi-stage decision making, fuzzy state and terminal regulators and their relationship to non-fuzzy quadratic state and terminal regulators," International Journal of Control, 17, 1973.
57. J. Kacprzyk, " Decision making in a fuzzy environment with fuzzy termination time," Fuzzy Sets and Systems, 1, 169-178, 1978.

58. H. Nojiri, "A model of fuzzy team decision," Fuzzy Sets and Systems, 2, 201-212, 1979.
59. W.E. Stein, "Optimal stopping in a fuzzy environment," Fuzzy Sets and Systems, 3, 253-259, 1980.
60. A.O. Esogbue and V. Ramesh, "Dynamic programming and fuzzy allocation processes," Tech. Memo. No. 202, Department of Operations Research, Case Western University, 1970.
61. J. F. Baldwin and B. W. Pilsworth, "Dynamic programming for fuzzy systems with fuzzy Environment", Journal of Mathematical Analysis and Applications, 85, 1-23, 1982.
62. Kacprzyk J., Staniewski P., "A new approach to the control of stochastic systems in a fuzzy environment", Arch. Autom. & Telemekh. XXV, 443-444, 1980.
63. G. Sommer, Fuzzy inventory scheduling, in Lasker (ed), 3052-3060, 1981.
64. Kacprzyk, J. and P. Staniewski, "Determination of long term inventory policies via fuzzy decision-making models", Proc. 5th Italian-Polish Symp. Appl. Syst. Anal. Econ. & Technol. Torun(Poland), 1980.

65. Kacprzyk, J. and P. Staniewski, "Application of fuzzy decising-models in inventory control problems" (in Russian). Proc. 7-th Bulgarian-Polish Symp.Anal. Konstancin (Poland), 1980.

66. Kacprzyk J. and P. staniewski, "Application of the theory of Markov potentials to the control of stochastic systems in a fuzzy environment"(in Polish). Proc. 8th Nat. Conf. Autom. Contr., Szczecin(Poland), 1980.

67. J. Kacprzyk and P. Staniewski, "A fuzzy approach to inventory control", Proc. 8th Syst. Sci. Conf., Wroclaw(Poland), 1981.

68. Kacprzyk, J. and P. Staniewski, "Long term inventory policy-making through fuzzy decision-making models", Fuzzy Sets and Systems 8, pp117-132, 1982.

69. Augustine O. Esogbue and Richard E. Bellman, "Fuzzy dynamic programming and its extensions", TIMS/Studies in the Management Sciences 20, 147-167, 1984.

70. Kacprzyk, J. and A. Straszak, "Application of fuzzy decision-making models for determining optimal policies in 'stable' integrated regional development", in P. P. Wang and S. K. Chang (Eds.): Fuzzy Set Theory and Application to Policy Analysis and Information Systems, Plenum, New

York, 1980.

71. Kacprzyk, J. and A. Straszak, "Determination of 'stable' integrated regional development trajectories via fuzzy decision-making models", Proc. 5th Italian-Polish Symp. Appl. Syst. Anal. Econ. & Technol., Torun (Poland), 1980.

72. Kacprzyk, J. and A. Strazak, "A fuzzy approach to the 'stability' of integrated regional development", in G. E. Lasker (Ed.): Proceedings of International Congress of Applied Systems Research and Cybernetics, Acapulco (Mexico), Pergamon Press, Toronto, 1982.

73. Kacprzyk, J. and A. Straszak, "Determination of 'stable' regional development trajectories via fuzzy decision-making model", in R. R. Yager (Ed.): Recent Advances in Fuzzy Set and Possibility Theory, Pergamon Press, New York, 1982.

74. A. O. Esogbue, "Dynamic Programming, Fuzzy Sets, and the Modeling of R&D Management Control System," IEEE Transactions on Systems, Man and Cybernetics, SMC - 13, 18-30, 1983.

75. Rinks, D.B., "The performance of fuzzy algorithm models for aggregate planning under differing cost structures,"

in: Gupta and Sanchez, (eds.), Fuzzy Information and Decision Processes, New York, North-Holland, 267-278, 1982.

76. Cynthia Susan McCahon, Fuzzy Set Theory Applied to Production and Inventory Control, Ph.D. Dissertation, Kansas State University, 1987.

77. Van Laarhoven, P. J. M. and W. Pedrycz, "A fuzzy extension of Saaty's prioritytheory," Fuzzy Sets and Systems, 11, 229-241, 1983.

78. R. Jain, "Decision-Making in the presence of fuzzy variables, IEEE. Trans. Systems Man Cybernet. 6, 698-703, 1976.

79. R.Jain, "A procedure for multiple-aspect decision-making using fuzzy sets", Internat. J. Systems Sci., 8, 1-7, 1977.

80. R. R. Yager, "Ranking fuzzy subsets over the unit interval", Proc. 1978 CDC, 1435-1437, 1978.

81. S. M. Baas and H. Kwakernaak, "Rating and ranking of multiple-aspect alternatives using fuzzy sets" Automatic, 13, 47-58, 1977.

82. J. F. Baldwin and N. C. F. Guild, "Comparison of fuzzy

sets on the same decision space", Fuzzy Sets and Systems, 2, 213-233, 1978.

83. W., Chang, "Ranking of fuzzy utilities with triangular membership function", Proc. Int. Conf. on Policy Anal. and Inf. Systems, 263-272, 1981.

84. D. Dubois and H. Prade, "Ranking of fuzzy numbers in the setting of possibility theory", Inform. Sci., 30, 183-224, 1983.

85. R. R. Yager, "A procedure for ordering fuzzy subsets of the unit interval", Inform. Sci., 24, 143-181, 1981.

86. E. E. Kerre, "The use of fuzzy set theory in electrocardiological diagnostics", in: M. M. Gupta and E. Scanchey, (Eds.), Approximate Reasoning in Decision Analysis, North-Holland, Amsterdam, 277-282, 1982.

87. Arnold Kaufman and Madan M. Gupta, Introduction to Fuzzy Arithmetic, Theory and Applications, Van Nostrand Reinhold Company Inc., 1985.

88. G. Bortolan and R. Degani, "A review of some methods for ranking fuzzy subsets", Fuzzy Sets and Systems 15, 1-19, 1985.

89. E. Stanley Lee and Rong-jun Li, "Comparison of fuzzy

numbers based on the probability measure of fuzzy events",
to be published.

90. Rong-jun Li and E. S. Lee, "Ranking fuzzy numbers---A
comparison", to be published.

91. Janusz Kacprzyk, Multistage Decision-Making Under
Fuzziness. Theory and Applications, Verlag TUV Rheinland,
1983.

MULTI-PERIOD PRODUCTION PLANNING IN FUZZY ENVIRONMENT

by

Yu-Long Liu

B.S., East-China Petroleum Institute, 1976

M.S., East-China Petroleum Institute, 1981

AN ABSTRACT OF A THESIS
submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

College of Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1989

Approved by:



Major Professor

MULTI-PERIOD PRODUCTION PLANNING IN FUZZY ENVIRONMENT

ABSTRACT

In the real-world multi-period production planning (MPPP) problems, the parameters must be estimated and they are frequently given by interval estimates. But for almost all current production planning models, these interval estimates must be translated into single numbers. This kind of translation often results in errors and in the loss of a considerable amount of information.

A general fuzzy approach by the use of fuzzy numbers was found to be an effective method in solving the interval MPPP problem. It is described as follows:

- (1) employing appropriate fuzzy numbers to represent the interval estimates in the multi-stage decision problems;
- (2) using the operations of fuzzy numbers combined with dynamic programming to solve the problem;
- (3) determining the required minimum or required maximum fuzzy number by the use of fuzzy number ranking techniques.

Three MPPP example problems with fuzzy costs and/or fuzzy demands were solved by this approach. The fuzzy representatives of schedule and minimum total cost is the main advantage of our fuzzy MPPP method. This kind of

solution can give the management clear ideas about the interval units of production at each period and the interval amounts of the required minimum total cost. Therefore, the management can manage the production flexibly and control the cost confidently.

This approach can be used to solve not only the problem with fuzzy goals and fuzzy constraints defined in the same space, but also the problem with fuzzy goals and constraints defined in different spaces. This advantage, in our view, will result in wide application of dynamic programming to fuzzy systems. Three other examples of fuzzy multi-stage decision making problems given in this thesis support this view.